1. (30) You enter a power plant which is burning the off-gas from a natural gas purification process (containing various C3 and C4 hydrocarbons). You notice that the air coming out of the compressor is at 400°C and 191 PSIG while the gas exiting the turbine to the exhaust stack is at 527 °C. Assume that the gas entering the plant is at one atmosphere and 27°C, room temperature. Use the attached H-S diagram for air. **Draw the steps for the process** and calculate the following:

A. What is the efficiency of the compressor?
B. What is the temperature coming out of the combustion process?
C. How much work is lost in this process?
D. What is the overall efficiency of the power plant?
E. What would be the overall plant efficiency if the compressor and compressor were 100% efficient?

Extra Credit (leave the following to the end): How would you estimate the heat of combustion of the hydrocarbon mixture and the efficiency of the combustion process?

**Answer:**
A. The efficiency of the compressor can be calculated using the H-S diagram with the two points that are specified: 400°C(673K) and 191 PSIG (1.4 Mpa) and inlet at 1 atm (0.1 MPa) and 27 °C (300K). The isentropic work would be 800-460 = 340kJ/kg while the actual work is 850-460= 390kJ/kg. The efficiency is thus 340/390 =87%

B. For the gas exiting the turbine at 527°C = 800K at 1 atm., we need to figure out the temperature into the turbine. If the turbine were isentropic, the temperature would have been 1500K and ΔH would be 800kJ/kg (1800-1000kJ/kg). Taking into account the efficiency, ΔH would be 697kJ/kg. This makes the temperature ~1400K

C. The lost work comes from the increases in entropy due to the actual increases in entropy in the turbine and compressor. For each process the increase in entropy is ~0.1kJ/kgK, a total of 0.2
1 kJ/kgK. This gives 60 kJ/kg at 300 K ambient
D. The overall efficiency of the plant would be 697-390/(1700-800) = 0.34: power out/heat in.
E. For the ideal process, the efficiency would be ~ 800-350/1800-1000 = 0.45 (assuming the temperatures are correct).

2. (30) The hydrocarbon employed in the above process needs to be transported as a liquid from its source to the power plant. It is to be liquefied in a simple Linde plant without any internal heat exchange (only heat exchange to the ambient atmosphere). Assume that the gas is propane and use the attached diagram for C\textsubscript{3}H\textsubscript{8}. The hydrocarbon flows into the liquefaction process at 2 atm and 300 °K at a rate of 0.25 m\textsuperscript{3}/s. It leaves the process at the same pressure. The compression ratio of the compressor is 25/1.

A. Calculate the rate of production of liquid propane (kg/hour).

B. What is the capacity (m\textsuperscript{3}/hour of hydrocarbon at room temperature) and the "ideal" horsepower required for the compressor?

Draw the process (as much as possible) on the attached diagram

Answer:
A. The production rate is fairly straight-forward as the rate at which the gas flows into the process is the rate of the production of liquid: 0.25 m\textsuperscript{3}/s at 2 atm and 300 K is equivalent to 0.50 m\textsuperscript{3}/s at 1 atm and 300 K, assuming an ideal gas. This can be converted to moles/s by dividing by 22.4 l/mole [1 m\textsuperscript{3} = 10\textsuperscript{6} cm\textsuperscript{3} = 10\textsuperscript{3} l]. 0.50 m\textsuperscript{3}/s equals 500/22.4 = 22.3 moles per second. For propane with a molecular weight (C\textsubscript{3}H\textsubscript{8} = 44), this rate of liquid production is 44 x 22.3 = 983 g/s. This becomes 0.983 x 3600 = 3539 kg/hr

B. The capacity of the compressor depends on the ratio of amount produced as a liquid to the fraction condensed in the process. The diagram shows that only 60 percent is liquified each pass. (Isenthalpic from 310 K and 5.0 MPa to 0.1 MPa). The capacity of the compressor is easily calculated as 0.25/0.6 = 0.42 m\textsuperscript{3}/s = 1500 m\textsuperscript{3}/h while the power required is 680-508 = 172 kJ/kg which gives 169 kJ/s = 225 hp.
3. (25) If the desired increase in pressure during a stage in a process is large, it is often more efficient (less total work is required) to compress a fluid in several stages. The efficiency depends on the intermediate pressures (from the first to the second... to the final pressure).

A. Express the work required to compress an ideal gas from an initial Pressure, \( P_0 \), to a final pressure, \( P_f \), as a function of the intermediate pressures (from the first to the second... to the final pressure).
Extra Credit 1: Discuss the influence of heating or cooling between the two stages.

ANSWER:
The work in terms of the pressure(s) can be expressed as:

For an intermediate pressure $P_i$ the total work becomes:

$$W_{1 \rightarrow i \rightarrow 2} = C_v T_1 \left( \frac{P_i}{P_f} \right)^{\gamma - 1} - 1 + C_v T_i \left( \frac{P_2}{P_i} \right)^{\gamma - 1}$$

without inter cooling this becomes (by substitution for $T_i$ $(P_i,P_1)$):

$$W_{1 \rightarrow i \rightarrow 2} = C_v T_1 \left( \frac{P_i}{P_f} \right)^{\gamma - 1} - 1 + \left( \frac{P_2}{P_i} \right)^{\gamma - 1}$$

$$= C_v T_1 \left( \frac{P_2}{P_f} \right)^{\gamma - 1} - 1$$

No surprise, there is no difference in the work required if the same (adiabatic) process line is followed by the second compression process. Graphically, the areas on a PV diagram would be the same. On a PH diagram the sum of the $\Delta H$'s for the two processes would be the same at the total $\Delta H$ without changing the $H$ of the intermediate pressure (by cooling).

If the gas had been intercooled back to the original Temperature:

$$W_{1 \rightarrow i \rightarrow 2} = C_v T_1 \left( \frac{P_i}{P_f} \right)^{\gamma - 1} - 1 + \left( \frac{P_2}{P_i} \right)^{\gamma - 1}$$

differentiating with respect to $P_i$ and setting equal to zero:

Thus, the ratios of $P_i/P_1 = P_2/P_i$ gives the least work... This is identical to the case where the work in each of the two steps is the same. Generalizing, the work shared by each step for $n$ steps should be the same in each step. It should also be obvious that the reducing the temperature significantly below the initial temperature will not be beneficial as the isotherm will give the least work between the two pressures (approaching a Carnot cycle).

4. (20) The process in problem 2 involves recycle of the same gaseous hydrocarbon. It would be possible to eliminate the recycle by cooling the propane below -25°C (at 2 Bar). This would require...
energy to provide refrigeration. Use the attached diagram for refrigerant 12 and determine the minimum energy (hp) required (per kg of gaseous propane) to liquify the propane employing this fluid (12) as the refrigerant (using an expansion valve and an ideal compressor) and air (300K) as the heat exchange medium.

Hint: The temperatures of the fluid in the two heat exchangers of the refrigerator are first estimated. Draw the diagram and estimate the COP. This can be related to the heat of liquifaction of propane with specific assumptions (state them).

Answer: You will need to have the refrigeration system operate from -35°C (238K) to 310K. For refrigerant 12, this means 0.08 MPa and 0.88 MPa.

The ideal compressor (isentropic) would take 40kJ/kg of refrigerant 12. The amount of heat removed is ~200-95 (if saturated liquid into the expansion valve) (this is correct) or 200-45 (if saturated liquid out of the expansion valve). This would give a COP of 105/40 = 2.625 [ or 155/40 = 3.875] . This gives the ratio heat removed to the work required.

The heat of liquifaction of propane at 2Bar is 400kJ/kg (from the diagram). Dividing this by the COP would give the amount of work required to liquify the propane assuming that the heat exchange is adiabatic (no exchange of heat with the ambient). Thus 152.3 kJ work/kg propane liquified. [103.2kJ/kg].