Problem 1  (35 points)

Three (3) gmoles of an ideal gas undergoes a closed cycle of four steps returning to its original state. The initial and final temperatures are 127 °C. Calculate $\Delta H$, $\Delta U$, $Q$, $W$ in Joules(±) and the final temperature for each step in the process. Note that this gas has $C_p = (5/3)R$ where $R = 8.3143$ Joules/mol-°C.

State A  
127°C, 3 gmol of gas

Step 1 (A→B)  
The gas is expanded at constant pressure until the volume is twice the initial volume.

Step 2 (B→C)  
The gas is further expanded at constant temperature until the volume is again doubled.

Step 3 (C→D)  
The gas is cooled at constant volume until:

Step 4 (D→A)  
The gas is compressed adiabatically to the original state A.

<table>
<thead>
<tr>
<th></th>
<th>Step 1 (A - B)</th>
<th>Step 2 (B - C)</th>
<th>Step 3 (C - D)</th>
<th>Step 4 (D - A)</th>
<th>Net Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work, W</td>
<td>-9976.8</td>
<td>-13830.8</td>
<td>0.00</td>
<td>5819.8</td>
<td>-17978.8</td>
</tr>
<tr>
<td>Heat, Q</td>
<td>16628</td>
<td>13830.8</td>
<td>-12471</td>
<td>0.00</td>
<td>17978.8</td>
</tr>
<tr>
<td>ΔU</td>
<td>6651.2</td>
<td>0.00</td>
<td>-12471</td>
<td>5819.8</td>
<td>0.00</td>
</tr>
<tr>
<td>ΔH</td>
<td>16628</td>
<td>0.00</td>
<td>-31177.5</td>
<td>14549.5</td>
<td>0.00</td>
</tr>
<tr>
<td>T(final)</td>
<td>800 °K</td>
<td>800 °K</td>
<td>50 °K</td>
<td>400 °K</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>$V_A \rightarrow 2V_A$</td>
<td>$2V_A \rightarrow 4V_A$</td>
<td>$4V_A$</td>
<td>$4V_A \rightarrow V_A$</td>
<td></td>
</tr>
</tbody>
</table>

Hint: First list what you know concerning each state in the sequence of steps. If you get stuck, consider the reverse of the process steps and what they imply.

Note that the initial temperature, $T_A$, is 127°C or 400 °K, that the total number of moles is 3 gmoles.
Problem 1 (cont'd)

Step 1 Constant P

\[ V_B = 2 \ V_A \text{ and } T_B = T_A \left( \frac{V_B}{V_A} \right) \] so that \( T_A = 400^\circ \text{K} \) so that \( T_B = 800^\circ \text{K} \).

then

\[ W = P_A (V_B - V_A) = NR (T_B - T_A) \] so that

\[ W = (3 \ \text{g moles})(8.314 \ \text{joules/mol} \cdot ^\circ \text{K})(800 - 400^\circ \text{K}) = 9976.8 \ \text{joules} \]

\[ Q = \Delta H = NC_p (T_B - T_A) \] since \( C_p = (5/3)R \), then \( Q = \Delta H = (5/3)W = 16628 \ \text{joules} \)

\[ \Delta U = NC_v (T_B - T_A) \] and since \( C_v = (2/3)R \), then \( \Delta U = (2/5)\Delta H = 6651.2 \ \text{joules} \)

Step 2 Constant T

\[ V_C = 2V_B \]

\[ -Q = W = NRT_B \ln \left( \frac{V_C}{V_B} \right) \] and \( Q = -W = (3)(8.314)(800)\ln(2) = 13830.8 \ \text{joules} \)

\[ \Delta U = \Delta H = 0 \] and \( T_C = T_B \) so that \( T_C = 800^\circ \text{K} \)

Step 3: Constant V

\[ V_D = V_C \] so that \( W = 0 \)

To calculate \( T_D \), one must calculate the initial temperature from step 4.

\[ \frac{T_D}{T_A} = \left( \frac{V_D}{V_A} \right)^{\gamma - 1} \] so here we have \( T_D = 400(1/4)^{1.5} = 50^\circ \text{K} \)

\[ Q = \Delta U = NC_v (T_D - T_C) \] then \( Q = (3)(2/3)(8.314)(50 - 800) = -12471 \ \text{joules} \)

\[ \Delta H =NC_p (T_D - T_C) = \gamma (\Delta U) = (5/2)(-20785) = -31177.5 \ \text{joules} \)

Step 4: Adiabatic

We saw that the initial temperature for this step was as follows:

\[ \frac{T_D}{T_A} = \left( \frac{V_D}{V_A} \right)^{\gamma - 1} \] so as above

\[ \Delta U = NC_v (T_A - T_D) = 3(2/3)(8.314)(400-50) = 5819.8 \ \text{joules} \]

\[ \Delta H =NC_p (T_A - T_D) = \gamma (\Delta U) = (5/2)(9699.67) = 14549.5 \ \text{joules} \]

\( Q = 0 \) and \( W = -\Delta U = -9699.67 \ \text{joules} \)

Extra: for the efficiency: \( \sum \Delta W_{\text{in}}/Q_{\text{in}} = 17987/30458 = 0.59 \)
Problem 3 (35 points)

In order to purify gas in the laboratory, one often will pass the gas over an activated carbon bed (an adsorption trap) at low temperature (e.g., at liquid nitrogen temperature, 77K). The impurities are trapped out on the activated carbon and some of the gas is also adsorbed. Following the purification, the trap is isolated from the system. It is crucial to realize that gas is still adsorbed on the activated carbon that will go into the gas phase as the trap warms up. I had a colleague that did not realize this, causing much trouble and excitement.

The activated carbon trap is typically ~ 1 liter containing 250 g of activated carbon. If you were purifying hydrogen, approximately 10 grams of hydrogen would be adsorbed during the purification which would then become gas if the trap is warmed to room temperature (assume 27°C). Assume you then placed this trap at room temperature into an insulated, sealed plastic container (a cooler) with a total volume of 2 liters [one liter of air at room temperature and one liter for the trap]. If the trap then breaks, the hydrogen then expands to fill your sealed container. Instantaneously, the temperature(s) change as does the pressure in the container. Assume that the hydrogen and air in the container behave as ideal gasses with Cp = 3 R and determine the instantaneous temperatures at the time of the break and the pressure within the container in order to see if the container will break or melt if the trap breaks. State all assumptions.

Hints: It might be useful to determine the final temperature and pressure after both thermal and mechanical equilibrium are achieved before you attempt to calculate the intermediate temperatures at the time of the break. Also, you can assume that there no heat transfer between the trap and the air in the container at the time of the break. R = 83.14 cm³ bar/gmoleK.
Answer: Following the hint, it should be obvious that $Q = W = 0$ for the overall process.

Thus, $\Delta U = 0$ and the final temperature is the same as the initial temperature $\rightarrow 27^\circ C$

Now we can calculate the final Pressure for the system from the ideal gas law.

$P = \frac{NRT}{V}$ where $N = N_1 + N_2 = 10/2 + 1*1000/83.14*300 = 5 + 0.04$

$P = 5.04*83.14*300/2000 = 63$ Bar also $P_1 = 125$ Bar in the trap (it may break)

The temperature calculation is a little more complex.

The expansion can be analyzed (see hint) as if it had occurred adiabatically

$$\left(\frac{T_1 f}{T_1 i}\right) = \left(\frac{P_1 f}{P_1 i}\right)^{\gamma - 1/\gamma}$$  ... $\gamma = 3/(3-1) = 1.5$

$T_1 f/300 = (1/2)^{1/3} = 0.7937$

$T_1 f = 238$ K

At this point $N_1 = 63*1000/83.14*238 = 3.18$ thus $N_2 = 1.856$

**Three approaches:**

A. The energy balance $n_1 \, Cv \, T_1 + n_2 \, Cv \, T_2 = n_1 \, Cv \, T_1 + n_2 \, Cv \, T_2$ final

This could be rearranged to give:

$300 * 5.04 = N_1 f \, T_1 f + N_2 f \, T_2 f$ in units of moles*K

$1512 = 3.18 * 238 + 1.86 \, T_2 f$

$T_2 f = (1512 - 756.8)/1.86 = 406$K

B. One can use the gas law

And $T = PV/RN = 63*1000/83.14*1.86 = 407$K

C. Another way:

There is a mass balance $N_2 i + N_1 i = N_2 f + N_1 f = 5.04$

The assumption here is that the pressure changes but then does not change as the temperatures equilibrate.

but with constant Volumes, initial Temp and Final Pressures:

$1/150 \left(\frac{1}{K}\right) = 1/T_1 + 1/T_2$

initial $P_1/T_1 + P_2/T_2 = 126/300$ bar/K= final $P_1/T_1 + P_2/T_2 = 63 \left(1/T_1 + 1/T_2\right)$

$1/150 \left(\frac{1}{K}\right) = 1/238 + 1/T_2$

$T_2 = 406$ K $= 133^\circ C$ (many plastics will melt)
3. The following diagram represents part of a process to extract power from a hot ammonia stream. The ammonia (NH₃, Mw=17) fluid enters a heat exchanger at 0.2kg/s and is cooled from 520°C to 200°C. The heat capacity of this fluid is given below. Within the process, water enters the heat exchanger as a saturated liquid at 190°C and 1.26 MPa pressure at a flow rate of 2.2 kg/s (point A). After vaporization (and superheating), it passes to a turbine entering at 1.26 MPa (point B). The steam exits the turbine at 0.2 MPa (point C). You can assume steam behaves as an ideal gas with constant Cp/R of 4.0 and a heat of vaporization of 1977kJ/kg for these calculations. Treat the heat exchanger and turbine as ideal with no loss of heat to the surroundings.

The heat capacity of the ammonia fluid is
\[ \frac{C_p}{R} = 22.6 + 0.1 T + 0.0002 T^2, \quad T \text{ in } K \]

*You are to estimate the power delivered by the turbine in horse power where 1hp = 0.7457kW and 1W = 1J/s.*

Hint: You need to estimate the temperatures at points B and C so that you can calculate the power delivered by the turbine.
Problem 3: First Exam

Heat exchanger

<table>
<thead>
<tr>
<th>Stream</th>
<th>Inlet T</th>
<th>Outlet T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ammonia</td>
<td>520</td>
<td>793K</td>
</tr>
<tr>
<td>Inlet</td>
<td>200</td>
<td>473K</td>
</tr>
<tr>
<td>(\Delta H) (J/s)</td>
<td>5,250,362 J/s</td>
<td></td>
</tr>
</tbody>
</table>

\[
0.2 \times 1000 \times \frac{8.314}{17} \times ((22.6 \times (C5 - C6) + (0.1/2) \times (C5^2 - C6^2) + (0.0002/3) \times (C5^3 - C6^3)))
\]

707,375 1,981,275 2,561,713

Water Stream 2.2 kg/s

| \(\Delta H_{vap}\) | 1977 | 4,349,400 J/s |
| \(\Delta H_{sensible}\) | 900,962 J/s |
| \(\Delta T\) | \(\frac{\Delta H}{nC_p}\) (J/s) | 222 |
| \(C_13/(C10*(1000/18)*4*8.314)\) | T (B) | Tin + \(\Delta T\) | 685K |
| \(\gamma = 4/3\) | T(C) | \(\frac{T_i(P/P_i)^R}{C_p}\) | 432K |

\(\Delta H_{turbine}\) \(= nC_p\Delta T\) 1,026,336 J/s

\(C10*4*8.314*(C16-C18)*1000/18\)
in HP 745.7 J/s/HP 1,376