Heat Exchangers - Introduction

Concentric Pipe Heat Exchange

Energy Balance on Cold Stream (differential)

\[ dQ_C = \left( wC_p \right)_C dT_C = C_c dT_C \]

Energy Balance on Hot Stream (differential)

\[ dQ_H = \left( wC_p \right)_H dT_H = C_H dT_H \]

Overall Energy Balance (differential)

For an adiabatic heat exchanger, the energy lost to the surroundings is zero so what is lost by one stream is gathered by the other.

\[ dQ_C + dQ_H = 0 \]
Heat Exchange Equation

It follows that the heat exchange from the hot to the cold is expressed in terms of the temperature difference between the two streams.

\[ dQ_H = U \left( T_H - T_C \right) dA \]

The proportionality constant is the “Overall” heat transfer coefficient (discussion later)

Solution of the Energy Balances

The Energy Balance on the two streams provides a delation for the differential temperature change.

\[ dT_H = \frac{dQ_H}{C_H} \quad \text{and} \quad dT_C = \frac{dQ_C}{C_C} \]

However, we should recall that we have an adiabatic heat exchanger so that

\[ d(\Delta T) = - \frac{dQ_H}{C_H} \left( 1 + \frac{C_H}{C_C} \right) \]

Overall Energy balances on each stream

Hot Fluid
\[ Q_H = C_H \left( T_{H1} - T_{H2} \right) \]

Cold fluid
\[ Q_C = C_C \left( T_{C2} - T_{C1} \right) \]

Overall Energy balance on the Exchanger
\[ Q_C + Q_H = 0 \]
The equation for $\Delta T$ can be modified using the overall energy balances to yield

$$d(\Delta T) = \frac{dQ_H}{C_H} \frac{\Delta T_2 - \Delta T_1}{(T_{H1} - T_{H2})}$$

The denominator is the energy lost by the hot stream, so

$$d(\Delta T) = \frac{dQ_H}{Q_H} (\Delta T_2 - \Delta T_1)$$

Application of the relation for energy transfer between the two streams yields

$$d(\Delta T) = -\frac{U dA \Delta T}{Q_H} (\Delta T_2 - \Delta T_1)$$

Integration of the relation is the basis of a design equation for a heat exchanger.

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \frac{UA}{Q_H} (\Delta T_2 - \Delta T_1)$$

Rearrangement of the equation leads to

The Design Equation for a Heat Exchanger

$$Q_H = UA \frac{(\Delta T_2 - \Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = UA\Delta T_{lm}$$
Design of a Parallel Tube Heat Exchanger

The Exchanger

The Design Equation for a Heat Exchanger

\[ Q_H = UA \left( \frac{\Delta T_2 - \Delta T_1}{\ln \left( \frac{\Delta T_2}{\Delta T_1} \right)} \right) = UA \Delta T_{lm} \]

Glycerin-water solution with a Pr = 50 (at 70 °C) flows through a set of parallel tubes that are plumbed between common headers. We must heat this liquid from 20 °C to 60°C with a uniform wall temperature of 100 °C. The flow rate, F, is 0.002 m³/sec (31.6 gal/sec).

- How many parallel tubes are required?
- How do we select L and D for these tubes?

Data
The heat capacity, \( C_p \), is 4.2 kJ/kg·°K
The density, \( \rho \), is 1100 kg/m³
The liquid has a kinematic viscosity, \( v = 10^{-3} \) cm²/sec.
Step 1
Calculate the heat load

\[ Q_c = \rho FC_p (T_{out} - T_{out}) \]

\[ Q_c = \left( 1100 \, \frac{\text{kg}}{m^3} \right) \left( 0.002 \, \frac{m^3}{\text{sec}} \right) \left( 4.2 \, \frac{\text{kJ}}{\text{kg} \cdot \text{°K}} \right) \frac{1^\circ \text{K}}{1^\circ \text{C}} (60 - 20)^\circ \text{C} \]

\[ Q_c = 369.6 \, \frac{\text{kJ}}{\text{sec}} = 369.6 \, \text{kWatts} \]

Step 2
Calculate the heat transfer coefficient

If the flow is laminar, likely since glycerin is quite viscous, and the Re < 2000
the Nusselt number relation for laminar flow can be expressed as

\[ \text{Nu} = \left[ (3.66)^3 + (1.61)^3 Gz \right]^{1/3} \]

The Graetz number is

\[ Gz = \text{Re Pr} \frac{D}{L} \]

If the flow is turbulent (Re > 2000), the Nusselt number is given by

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \]

We do not know the flow per tube and therefore we do not know the Re.
However we don’t need to know that. In Lecture 27 we observed for Heat Transfer in a Tube that

\[ \frac{T - T_R}{T_1 - T_R} = \exp \left( - \frac{\pi Dhz}{\omega C_p} \right) = \exp \left( -4St \frac{z}{D} \right) \]
The definition of the Stanton Number is:

\[ St = \frac{h}{\rho C_p U} = \frac{Nu}{RePr} = \frac{Nu}{Pe} \]

Given a Re and Pr, we can calculate the Nu and the Stanton Number, the latter providing us with the temperature at length L from the previous equation. Let’s examine several configurations at \( L/D = 50, 100, 200 \). The Excel table below can be used to specify a design chart.

<table>
<thead>
<tr>
<th>Pr = 50</th>
<th>L/D = 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>Nu</td>
</tr>
<tr>
<td>1</td>
<td>3.7610</td>
</tr>
<tr>
<td>3</td>
<td>3.9482</td>
</tr>
<tr>
<td>6</td>
<td>4.1996</td>
</tr>
<tr>
<td>10</td>
<td>4.4940</td>
</tr>
<tr>
<td>20</td>
<td>5.0980</td>
</tr>
<tr>
<td>30</td>
<td>5.5852</td>
</tr>
<tr>
<td>100</td>
<td>7.7548</td>
</tr>
<tr>
<td>200</td>
<td>9.5962</td>
</tr>
<tr>
<td>500</td>
<td>12.8779</td>
</tr>
<tr>
<td>1000</td>
<td>16.1628</td>
</tr>
<tr>
<td>2000</td>
<td>20.3244</td>
</tr>
<tr>
<td>5000</td>
<td>100.1133</td>
</tr>
<tr>
<td>10000</td>
<td>174.3074</td>
</tr>
<tr>
<td>20000</td>
<td>303.4868</td>
</tr>
<tr>
<td>30000</td>
<td>419.7714</td>
</tr>
</tbody>
</table>

To obtain the numbers in the spreadsheet, we used the Nusselt number relation for laminar flow expressed as

\[ Nu = \left[ \left( 3.66 \right)^3 + \left( 1.61 \right)^3 Gz \right]^{1/3} \]
and for turbulent flow as

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \]
Step 3  
Calculate the Area required

Base case  
D = 2 cm. and L = 100 D = 2 meters

For this case we observe that from the calculations for $\theta_{cm}$

<table>
<thead>
<tr>
<th>Re</th>
<th>L/D = 50</th>
<th>L/D = 100</th>
<th>L/D = 200</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0233</td>
<td>0.0006</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.2682</td>
<td>0.0789</td>
<td>0.0069</td>
</tr>
<tr>
<td>8.8</td>
<td>0.5000</td>
<td>0.3966</td>
<td>0.1718</td>
</tr>
<tr>
<td>10</td>
<td>0.6380</td>
<td>0.4387</td>
<td>0.2099</td>
</tr>
<tr>
<td>12</td>
<td>0.6800</td>
<td>0.4966</td>
<td>0.2682</td>
</tr>
<tr>
<td>12.3</td>
<td>0.6854</td>
<td>0.5042</td>
<td>0.2763</td>
</tr>
<tr>
<td>24.4</td>
<td>0.8040</td>
<td>0.6836</td>
<td>0.5017</td>
</tr>
<tr>
<td>50</td>
<td>0.8805</td>
<td>0.8073</td>
<td>0.6888</td>
</tr>
<tr>
<td>60</td>
<td>0.8945</td>
<td>0.8301</td>
<td>0.7254</td>
</tr>
<tr>
<td>70</td>
<td>0.9050</td>
<td>0.8473</td>
<td>0.7532</td>
</tr>
<tr>
<td>100</td>
<td>0.9254</td>
<td>0.8805</td>
<td>0.8073</td>
</tr>
<tr>
<td>200</td>
<td>0.9532</td>
<td>0.9254</td>
<td>0.8805</td>
</tr>
<tr>
<td>500</td>
<td>0.9746</td>
<td>0.9596</td>
<td>0.9358</td>
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<tr>
<td>1000</td>
<td>0.9840</td>
<td>0.9746</td>
<td>0.9596</td>
</tr>
<tr>
<td>2000</td>
<td>0.9899</td>
<td>0.9840</td>
<td>0.9746</td>
</tr>
<tr>
<td>5000</td>
<td>0.9802</td>
<td>0.9913</td>
<td>0.9862</td>
</tr>
<tr>
<td>6000</td>
<td>0.9809</td>
<td>0.9923</td>
<td>0.9878</td>
</tr>
<tr>
<td>8000</td>
<td>0.9819</td>
<td>0.9936</td>
<td>0.9899</td>
</tr>
<tr>
<td>9000</td>
<td>0.9824</td>
<td>0.9941</td>
<td>0.9907</td>
</tr>
</tbody>
</table>

We can observe that the flow rate per tube is given by

$$F_{nt} = \frac{F}{n_t}$$

so that the Reynolds’ number is

$$Re = \frac{4F}{\pi D u n_t}$$
As a consequence we can observe that the total length of tubing is not dependent on \( D \) alone but on other considerations that might set a condition for \( \text{Re} \), e.g. a pressure drop limitation. We find that for this base case, we find

\[
n_tL = A = \frac{4F}{\pi \nu \text{Re}} \frac{L}{D}
\]

We find that \( \Theta_{cm} = 0.5 \)

<table>
<thead>
<tr>
<th>( L/D )</th>
<th>( \text{Re} )</th>
<th>( n_tL )</th>
<th>( n_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>8.8</td>
<td>14.47</td>
<td>14.46</td>
</tr>
<tr>
<td>100</td>
<td>12.3</td>
<td>20.70</td>
<td>10.35</td>
</tr>
<tr>
<td>200</td>
<td>24.4</td>
<td>20.87</td>
<td>5.21</td>
</tr>
</tbody>
</table>

Does it make sense?
Maximum Cooling Capacity of an Exchanger of Fixed Area

Water is available for use as a coolant for an oil stream in a double-pipe heat exchanger.
  - The flow rate of the water is 500 lb/hr.
  - The heat exchanger has an area of 15 ft².
  - The oil heat capacity, \( C_{po} \), is 0.5 BTU/lb-°F
  - The overall heat transfer coefficient, \( U \), is 50 BTU/hr-ft²-°F

The initial temperature of the water, \( T_{w0} \), is 100°F
The maximum temperature of the water is 210°F
The initial temperature of the oil, \( T_{w0} \), is 250°F
The minimum temperature of the oil, \( T_{w0} \), is 140°F

Estimate the maximum flow rate of oil that may be cooled assuming a fixed flow rate of water at 500 lb/hr

There are two possible modes of operation
  - Co-current flow
  - Counter-current flow
Let us look at both cases

Co-current flow

Constraints
  \[ T_w < 210; \quad T_w < T_o; \quad T_o \geq 140 \]

Energy balances
  - Oil
    \[ Q_o = F_o C_{po}(T_{o1} - T_{o2}) = F_o(0.5)(250 - T_{o2}) \]

  - Water
    \[ Q_w = F_w C_{pw}(T_{w1} - T_{w2}) \]
    \[ F_o C_{po}(T_{o1} - T_{o2}) = 500(1.0)(210 - 100) = 55,000 \text{ BTU/hr} \]
Recall the Design equation

\[ Q_H = UA \frac{(\Delta T_2 - \Delta T_1)}{\ln \left( \frac{\Delta T_2}{\Delta T_1} \right)} = UA \Delta T_{lm} \]

Now the \( \Delta T_{lm} \) is given by

\[ \Delta T_{lm} = \frac{(\Delta T_2 - \Delta T_1)}{\ln \left( \frac{\Delta T_2}{\Delta T_1} \right)} = \frac{Q_w}{UA} = \frac{55000}{(50)(15)} = 73.3 \]

Using the temperatures, we obtain \( T_{0\text{max}} = 238.5 \) °F and from the heat balance for oil, we obtain

\[ F_o = \frac{C_{po}(T_{01} - T_{02})}{Q_o} = \frac{(0.5)(250 - 238.5)}{55000} = 9560 \text{ lb / h} \]

**Counter-current Flow**

**Constraints**

\( T_w < 210 ; T_w < T_o ; T_o \geq 140 \)

**Energy balances**

**Oil**

\[ Q_o = F_o C_{po}(T_{o1} - T_{o2}) = F_o(0.5)(250 - T_{o2}) \]

**Water**

\[ Q_w = F_w C_{pw}(T_{w1} - T_{w2}) \]
\[ F_o C_{p0} (T_{o1} - T_{o2}) = 500(1.0)(210 - 100) = 55,000 \text{ BTU / hr} \]

Recall the Design equation

\[ Q_H = UA \left( \frac{\Delta T_2 - \Delta T_1}{\ln(\frac{\Delta T_2}{\Delta T_1})} \right) = UA \Delta T_{lm} \]

Now the \( \Delta T_{lm} \) is given by

\[ \Delta T_{lm} = \frac{\left( \Delta T_2 - \Delta T_1 \right)}{\ln(\frac{\Delta T_2}{\Delta T_1})} = \frac{Q_w}{UA} = \frac{55000}{(50)(15)} = 73.3 \]

Using the temperatures, we obtain \( T_{0\text{max}} = 221 \) °F

and from the heat balance for oil, we obtain the oil flow rate as 3800 lbm/hr.

I thought that countercurrent flow was supposed to be more efficient. What is the problem?