Condensation and the Nusselt's Film Theory

Condensation is a rather complicated process. It was Wilhelm Nusselt's idea to reduce the complexity of the real process to a rather simple model, namely that the only resistance for the removal of the heat released during condensation occurs in the condensate film. The following gives an explanation of the Nusselt theory at the example of condensation on a vertical wall.

Condensation occurs if a vapor is cooled below its (pressure dependent) saturation temperature. The heat of evaporation which is released during condensation must be removed by heat transfer, e.g. at a cooled wall. Figure 1 shows how saturated vapor at temperature $T_s$ is condensing on a vertical wall whose temperature $T_w$ is constant and lower than the saturation temperature.

A condensate film develops which flows downwards under the influence of gravity. As condensation occurs over the whole surface the thickness of the film increases.
For laminar film flow heat can be transferred from the film surface to the wall only by heat conduction through the film (Figure 2).

![Figure 2](image)

The local heat flux at position $z$ through the film due to conduction is

$$q_z = \frac{k}{\delta(z)}(T_s - T_w)$$

where $k$ is the thermal conductivity of the condensate (which is assumed to be constant) and $\delta(z)$ is the film thickness at position $z$.

From the definition of the local heat transfer coefficient $h_{loc}$,

$$q_z = h_{loc}(T_s - T_w)$$

it follows that

$$h_{loc} = \frac{k}{\delta(z)}$$

The problem is reduced to the calculation of the film thickness profile. If $\delta(z)$ is known integration of equations (1) and (3) over the whole surface yields the total heat flow and the mean heat transfer coefficient.
The first step in determining w(y) is the calculation of the velocity profile w(y) in the condensate film (see Figure 3).

\[ w(y) \text{ can either be determined by applying the Navier-Stokes equation or directly from a force balance for a fluid element in the film (force exerted by the shear stress equals force of gravity minus buoyancy)} \]

\[ \mu \frac{d^2 w}{dy^2} = -(\rho - \rho_v)g \]

\[ \left. \frac{dw}{dy} \right|_{y=\delta} = 0 \ ; \ w(0) = 0 \]

and the solution is

\[ w(y) = -\frac{(\rho - \rho_v)g}{\mu} \left( \delta y - \frac{y^2}{2} \right) \]

Using the velocity profile equation (5) we can now calculate the condensate mass flow rate by integrating from y=0 to y=\( \delta \):

The result is:

\[ \dot{m} = \frac{\rho(\rho - \rho_v)gB}{\mu} \frac{\delta^3}{3} \]
By differentiating equation (6) we can also determine the change in the mass flow rate with the film thickness:

\[
\frac{d\dot{m}}{d\delta} = \frac{\rho(\rho - \rho_v)gB}{\mu} \delta^2
\]

The change of the condensate mass flow rate results from the condensation of vapor and requires the heat flow

\[
d\dot{Q} = \Delta H_v \, d\dot{m} = qBdz
\]

to be removed (\(\Delta H_v = \) enthalpy of evaporation). Using equations (1) and (7) the differential equation for the film thickness as a function of the coordinate \(z\) is:

\[
\delta^2 \frac{d\delta}{dz} = \frac{k \mu}{\rho(\rho - \rho_v)g} \left( T_s - T_w \right)
\]

Integration of equation (8) with the boundary condition \(\delta(0) = 0\), yields

\[
\delta = \left[ \frac{k \mu \left( T_s - T_w \right)}{\Delta H_v \rho (\rho - \rho_v)g} \right]^{1/4}
\]

The film thickness increases with the fourth root of the coordinate \(z\).

By substituting \(\delta\), according to equation (9) into equation (3) the local heat transfer coefficient follows:

\[
h_{loc} = \frac{k}{\delta} = \left[ \frac{\Delta H_v \rho (\rho - \rho_v)gk^3}{4\mu \left( T_s - T_w \right)z} \right]^{1/4}
\]
Finally, the mean heat transfer coefficient for a wall of height $L$ can be calculated by integrating the local heat transfer coefficient, $h_{loc}$, from $z = 0$ to $z = L$:

$$h_m = \frac{1}{L} \int_0^L h_{loc} \, dz = 0.943 \left[ \frac{\Delta H_v \rho (\rho - \rho_v) g k^3}{4 \mu (T_s - T_w) L} \right]^{1/4}$$

As we can see from this equation, the heat transfer coefficients are large for small temperature differences $t_s - t_w$ and heights $L$. In both cases the condensate film is thin and hence the heat transfer resistance is low.

Equation (11) can also be used for film condensation at the inner or outer walls of vertical tubes if the tube diameter is large compared to the film thickness. All fluid properties in equation (11) with the exception of the vapor density are best evaluated at the mean temperature

$$T_m = \frac{3}{4} T_w + \frac{3}{4} T_s$$

$\rho_v$ is evaluated at the saturation temperature $T_s$.

Nusselt derived a similar equation for film condensation on horizontal tubes using a numerical integration. The mean heat transfer coefficient for a single horizontal tube of diameter $D$ is

$$h_m = 0.728 \left[ \frac{\Delta H_v \rho (\rho - \rho_v) g k^3}{\mu (T_s - T_w) D} \right]^{1/4}$$
In spite of the simplifications heat transfer coefficients from the Nusselt theory are surprising accurate. Measured heat transfer coefficients are up to +25% higher than the values calculated from above equations. The main reason for the deviation is the formation of waves on the film surface which isn't considered in the Nusselt theory. These waves lead to an improvement in the heat transfer. Whitaker recommends for the rippling falling condensate film a value 20% larger so that we might use

\[ h_m = 1.137 \left( \frac{\Delta H_v \rho (\rho - \rho_v) g k^3}{\mu (T_s - T_w) L} \right) \]

The flow in the film could become turbulent at \( Re > 1800 \), in which case in the coefficient might be represented as

\[ Re = \frac{4}{3} \frac{g}{v^2} \left[ \frac{4v k (T_s - T_w) L}{\Delta H_v \rho g} \right] \]

so that the heat transfer coefficient is

\[ h_m = 0.0076 \left( \frac{\rho (\rho - \rho_v) g}{\mu^2} \right)^{1/3} Re^{-0.4} \]