CHE 333

Solutions to Problem Set 1

Problem 1.2

The conductivity, surface area and thickness of a wall separating a room from the ambient, are known. Given the temperature in the room, the heat loss can be described by 1-D conduction through the wall. The given values are used to set up the heat loss rate $Q$ by Fourier's law (see below).

$$k := 0.75, 1.0, 1.25 \quad Ti := 25 \quad To := -15, -10, 0, 10, 25, 40$$

$$S := 20 \quad L := 0.3$$

$$Q(To, k) := -k \cdot S \cdot \frac{(To - Ti)}{L}$$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$Q(To, 0.75)$</th>
<th>$Q(To, 1.0)$</th>
<th>$Q(To, 1.25)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75</td>
<td>$2.10^{-3}$</td>
<td>$2.667 \cdot 10^{-3}$</td>
<td>$3.333 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>1.5</td>
<td>$2.333 \cdot 10^{-3}$</td>
<td>$2.333 \cdot 10^{-3}$</td>
<td>$2.917 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>1.25</td>
<td>$2.083 \cdot 10^{-3}$</td>
<td>$1.667 \cdot 10^{-3}$</td>
<td>$1.667 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>1</td>
<td>$1.667 \cdot 10^{-3}$</td>
<td>$1.333 \cdot 10^{-3}$</td>
<td>$1.333 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>0.75</td>
<td>$1.25 \cdot 10^{-3}$</td>
<td>$1.0 \cdot 10^{-3}$</td>
<td>$1.0 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$833 \cdot 10^{-3}$</td>
<td>$666.667$</td>
<td>$666.667$</td>
</tr>
<tr>
<td>0</td>
<td>$416.667$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>-0.25</td>
<td>$-416.667$</td>
<td>$-333.333$</td>
<td>$-333.333$</td>
</tr>
<tr>
<td>-0.5</td>
<td>$-833 \cdot 10^{-3}$</td>
<td>$-666.667$</td>
<td>$-666.667$</td>
</tr>
<tr>
<td>-0.75</td>
<td>$-1.0 \cdot 10^{-3}$</td>
<td>$-1.0 \cdot 10^{-3}$</td>
<td>$-1.25 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

The heat loss rate is always higher for a wall of higher conductivity. All the curves meet at $To=25$ C where there is no heat loss (the room temperature = ambient temperature). The sign of $Q$ changes.
where there is no heat loss (the room temperature = ambient temperature). The sign of $Q$ changes as one crosses 25°C, since below this temperature the room loses heat, whereas it gains heat if the ambient temp is higher than 25°C.
Problem 1.6

This problem involves comparing the properties of two different materials for the same temperature difference. For the composite wall:

\[ q_c = -k_c \Delta T/L_c \]

For the masonry wall:

\[ q_m = -k_m \Delta T/L_m \] (for the same temperature difference)

It is required that \( q_m = 0.8 q_c \)

Hence \( -0.8 k_c \Delta T/L_c = -k_m \Delta T/L_m \)

Or

\[
\begin{align*}
    k_c :&= 0.25 \\
    L_c :&= 0.1 \\
    k_m :&= 0.75 \\
    \frac{k_m \cdot L_c}{0.8 \cdot k_c} :&= L_m = 0.375 \quad \text{m}
\end{align*}
\]

Obviously, since the masonry wall is more conducting, it will have to be thicker than the composite wall if one expects it to conduct only 80% of the heat conducted by the composite wall for the same temperature difference across them.
Problem 1.7

Another problem of 1-D conduction, this time across a chip that is 1 mm thick and with a surface area \( S = 5 \text{ mm square}. \) The heat generated by circuits in the chip is given and must be dissipated at the same rate to maintain steady-state.

\[
\begin{align*}
\kappa & = 150 \\
\tau & = 0.001 \\
S & = (0.005)^2 \quad \text{or} \quad S = 2.5 \times 10^{-5} \\
Q & = 4
\end{align*}
\]

Heat loss rate \( Q = kS \frac{\Delta T}{t} \Rightarrow DT = Q \frac{1}{kS} \)

So \( DT = 1.067 \text{ K} \)
Problem 1.13

This time the heat loss is described by a heat transfer coefficient (i.e., "Newton's law of cooling"). The heat loss rate \( Q = h S \Delta T \) where \( h \) is the heat transfer coefficient, \( S \) = surface area available for heat transfer, and \( \Delta T \) is the temperature difference. The maximum chip power allowed is determined by the heat transfer coefficient, since all this generated heat must be removed by the coolant.

So \( Q_{\text{max}} = h \Delta T S \)

For air cooling:

\( h_1 = 200 \)

\( \Delta T = 85 - 15 \quad \text{or} \quad \Delta T = 70 \)

\( S = (0.005)^2 \quad \text{or} \quad S = 2.5 \times 10^{-5} \)

\( Q_{\text{max}} = h_1 \cdot \Delta T \cdot S \)

\( Q_{\text{max}} = 0.35 \text{W} \)

Similarly for the dielectric coolant:

\( h_2 = 3000 \)

\( Q_{\text{max}} = h_2 \cdot \Delta T \cdot S \quad \text{or} \quad Q_{\text{max}} = 5.25 \text{W} \)

The higher the value of \( h \), the higher the maximum allowed chip power.
Problem 1.29

Water is used for heating a house. The daily water consumption (in volume units) is given, as also the temperature difference to be achieved (55-15 =40 deg. C). So the energy requirement is simply \( E = mC_p \Delta T \) where \( C_p \) is the specific heat of water.

\[
m := 1000 \cdot 100 \cdot \frac{365}{264.17} \quad \text{or} \quad m = 1.382 \cdot 10^5 \ \text{kg year}^{-1}
\]

\[
C_p := 4180 \ \text{J kg}^{-1} \text{K}^{-1}
\]

\[
\Delta T := 40
\]

\[
E := m \cdot C_p \cdot \Delta T \quad \text{or} \quad E = 2.31 \cdot 10^{10} \ \text{J year}^{-1}
\]

But 1 kWh = 3.6 * 10^6 J ==> \( E = 6417 \text{ kWh/ year} \)

If an electrical heater is used to directly heat the water, then assuming 100% efficiency and given the cost \( C \) of electrical power, the heating cost \( H \) will be:

\[
C := 0.08
\]

\[
H := 6417 \cdot C \quad \text{or} \quad H = 513.36 \ \text{dollars year}^{-1}
\]

Alternatively, a heat pump may be used. This device comprises a compressor that takes \( W \) units of work from an external source (e.g., electrical power), to circulate compressed refrigerant in the system. The refrigerant takes an amount of heat \( Q_c \) from the ground and releases heat \( Q_h \) in the house to heat the water. The energy balance on the heat pump gives

\[
W = Q_h - Q_c \quad (1)
\]

\( Q_h \) is known since it is the heat requirement (calculated earlier, 6417 kWh). Also the COP (coefficient of performance) of the heat pump is given.

\[
\text{COP} = \frac{Q_c}{W} \ , \ i.e., \ the \ amount \ of \ heat \ it \ can \ take \ in \ per \ unit \ external \ work \ provided. \ In \ this \ case, \ the \ COP =3 \Rightarrow Q_c = 3W \quad (2)
\]

(1) and (2) give \( W = C \cdot W = 1604.25 \text{ kWh} \)

However this work requires electrical power and the efficiency with which the electrical power is converted into work, is 85%. So the electrical power requirement will be \( W/0.85 = 1887.35 \text{ kWh} \)

Hence the cost \( H = 1887.35 \cdot 0.08 = 151 \ \text{dollars/year} \)

The heat pump is more economical since it takes part of the energy requirements from the surroundings.
Problem 1.35

Liquid oxygen is stored in a spherical tank of known size. Because of convective and radiative heat transfer from the surroundings, the oxygen evaporates; and the rate of evaporation is to be evaluated.

\[ h := 10 \quad To := 298 \]
\[ Ti := 263 \]

Emissivity \( e := 0.2 \)

Stefan constant \( s := 5.67 \times 10^{-8} \)

Flux \( q := h \cdot (To - Ti) + e \cdot s \cdot (To^4 - Ti^4) \)

or \( q = 385.174 \)

\( d := 0.5 \) (tank diameter)

The heat loss rate is
\[ Q := q \cdot 3.1416 \cdot d^2 \quad \text{or} \quad Q = 302.516 \frac{J}{s} \]

The latent heat of vaporization \( H \) is given, so the evaporation rate is:

\[ H := 214 \times 10^{-3} \]

\[ m := \frac{Q}{H} \quad \text{or} \quad m = 1.414 \times 10^{-3} \frac{kg}{s} \]

The expression for the flux shows that the evaporation rate is a linear function of the emissivity.

\( e := 0.2, 0.3 \ldots 1 \)

\[ m(e) := [h \cdot (To - Ti) + e \cdot s \cdot (To^4 - Ti^4)] \cdot 3.1416 \frac{d^2}{H} \frac{kg}{s} \]

End