ECE 673/CA770A: Homework 4

Due:
March 28 (On-Campus Students);
One week after watching Lecture 12 (Off-campus students).

(1) You are given an $M/M/1$ queue with finite queue size. At most $K$ packets can be stored in this queue: if any packets arrive when the queue is full, they are rejected. Suppose the arrival rate is $\lambda = 1$ and the service rate is $\mu = 1.5$. Then, plot the expected number of rejected jobs per second as a function of $K$, for $K = 0, 1, \cdots, 20$. (Use Markov chains, not simulation for your answer).

(2) In this problem, we will use the $\chi^2$ test. Consider the random number generator $X_n = 65539X_{n-1} \mod 2^{31}$. (Don’t use this generator in any of your simulations: it fails the spectral test very badly). Pick some seed value $X_0$ (don’t use $X_0 = 0!!$), generate a total of 5,000 random numbers and use “bins” of length 0.01 each from 0 to 1. (Since there are 100 bins in all, the number of degrees of freedom is 99).

You can find tables of the $\chi^2$ distribution online; for example in

Determine, using the $\chi^2$ test whether we should reject the generator as producing a uniformly distributed number over $[0, 1]$ if the p-value is 0.05.

Note: If $2^{31}$ is greater than the upper limit for integers permitted by your computer, use the largest power of 2 that can be represented as an integer.

(3) Repeat the above problem using the Kolmogorov-Smirnov test. If sample sizes are large, the asymptotic formula for KS can be used. At a significance level of 0.05, the critical value (called D in Ross) for the KS test is given by $1.36/\sqrt{n}$, where $n$ is the number of data points.

(4) Consider a single-server system. Job service times are exponentially distributed with mean $1/\mu$. Jobs arrive in pairs: each arrival consists of two jobs. These pairs arrive according to a Poisson process with rate $\lambda$. Draw the Markov chain associated with this process, and find the steady-state probability distribution. (Assume that $\lambda$ is small enough with respect to $\mu$ that steady-state probabilities exist).

(5) You have a five-processor system. Failure occur as Poisson processes with rate $\lambda$. You have one repairman, who can work on one processor at a time; it takes an exponentially distributed amount of time, with mean $1/\mu$ to repair one processor. Find the steady-state probability that there will be $n$ functional processors in the system.