ECE 697C/AD721A Sequential and Parallel Algorithms

Homework 4

Due within 10 days of watching Lecture 20

Exercise 23.2-7. The diameter of a tree, \( T = (V,E) \), is given by \( \max_{u,v \in V} \delta(u,v) \). That is, the diameter is the greatest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm.

Problem 23.3. An Euler tour of a connected, directed graph, \( G = (V,E) \), is a cycle that traverses each edge of \( G \) exactly once, although it may visit a vertex more than once.

- (a) Show that \( G \) has an Euler tour if and only if \( \text{indegree}(v) = \text{outdegree}(v) \) for each vertex \( v \in V \).
- (b) Describe an \( O(E) \)-time algorithm to find an Euler tour of \( G \) if one exists. 
  (Hint: Merge edge-disjoint cycles.)

Exercise 24.1-5. Let \( e \) be a maximum-weight edge on some cycle of \( G = (V,E) \). Prove that there is a minimum spanning tree of \( G' = (V,E - \{e\}) \) that is also a minimum spanning tree of \( G \).

Exercise 24.1-6. Show that a graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique light edge crossing the cut. Show that the converse is not true by giving a counterexample.

Exercise 26.2-6. Another way to reconstruct shortest paths in the Floyd-Warshall algorithm uses values \( \phi^{(k)}_{ij} \) for \( i,j,k = 1,2,\ldots,n \), where \( \phi^{(k)}_{ij} \) is the highest-numbered intermediate vertex of a shortest path from \( i \) to \( j \). Give a recursive formulation for \( \phi^{(k)}_{ij} \), modify the FLOYD-WARSHALL procedure to compute the \( \phi^{(k)}_{ij} \) values, and rewrite the PRINT-ALL-PAIRS-SHORTEST-PATH procedure to take the matrix \( \Phi = \left( \phi^{(k)}_{ij} \right) \) as an input.

Exercise 27.2-8. Show that a maximum flow in a network \( G = (V,E) \) can always be found by a sequence of at most \( |E| \) augmenting paths. (Hint: Determine the paths after finding the maximum flow).