Sequential and Parallel Algorithms

Homework 2

Due within 10 days of watching Lecture 8.

Exercises 12.2-6. Show that if |U| > nm, there is a subset of U of size n consisting of keys that all hash to the same slot, so that the worst-case searching time for hashing with chaining is \( \Theta(n) \).

Exercise 12.3-5. Show that if we restrict each component \( a_i \) of \( a \) in the equation

\[
\begin{align*}
    h_a(x) &= \sum_{i=0}^{r} a_i x_i \mod m
\end{align*}
\]

to be nonzero, then the set \( \mathcal{H} = \{h_a\} \) as defined in the equation

\[
\begin{align*}
    \mathcal{H} &= \bigcup_a [h_a]
\end{align*}
\]

is not universal. \((\text{Hint: Consider the keys } x = 0 \text{ and } y = 1)\).

Problem 12.3. Suppose that we have a hash table with \( n \) slots, with collisions resolved by chaining, and suppose that \( n \) keys are inserted into the table. Each key is equally likely to be hashed to each slot. Let \( M \) be the maximum number of keys in any slot after all the keys have been inserted. Prove an \( O(\lg n/\lg \lg n) \) upper bound on \( E[M] \), the expected value of \( M \).

- (a) Argue that the probability \( Q_k \) that \( k \) keys hash to a particular slot is given by

\[
    Q_k = \binom{n}{k} \left( \frac{1}{n} \right)^k \left( 1 - \frac{1}{n} \right)^{n-k}
\]

- (b) Let \( P_k \) be the probability that \( M = k \), i.e., the probability that the slot containing the most keys contains \( k \) keys. Show that \( P_k \leq nQ_k \).

- (c) Use Stirling’s approximation to show that \( Q_k < e^k/k! \).

Exercise 13.3-3. We can sort a given set of \( n \) numbers by first building a binary search tree containing these numbers (using TREE-INSERT repeatedly to insert the numbers
one by one) and then printing the numbers by an inorder tree walk. What are the
worst-case and the best-case running times for this sorting algorithm?

Exercise 13.3-4. Show that if a node in a binary search tree has two children, then its
successor has no left child and its predecessor has no right child.

Exercise 14.2-4. Let \( a, b, c \) be arbitrary nodes in subtrees \( \alpha, \beta, \gamma \), respectively, in the
tree shown below. How do the depths of \( a, b, c \) change when a left rotation is performed
on node \( x \)?