Sequential and Parallel Algorithms

Homework 1
Due within 10 days of watching Lecture 5

The following are from Cormen, Leiserson and Rivest (1990 edition).

Exercise 2.2-5. Is the function \(|\log n|!\) polynomially bounded? Is the function \(|\log \log n|!\) polynomially bounded?

Problem 2-4. Let \(f(n)\) and \(g(n)\) be asymptotically positive functions. Prove or disprove each of the following conjectures:

- (a) \(f(n) = O(g(n))\) implies \(g(n) = O(f(n))\).
- (b) \(f(n) + g(n) = \Theta(\min(f(n), g(n)))\).
- (c) \(f(n) = O(g(n))\) implies \(\log f(n) = O(\log g(n))\), where \(\log g(n) > 0\) and \(f(n) \geq 1\) for all sufficiently large \(n\).
- (d) \(f(n) = O(g(n))\) implies \(2^{f(n)} = O(2^{g(n)})\).
- (e) \(f(n) = O((f(n))^2)\).
- (f) \(f(n) = O(g(n))\) implies \(g(n) = \Omega(f(n))\).
- (g) \(f(n) = \Theta(f(n/2))\).
- (h) \(f(n) + o(f(n)) = \Theta(f(n))\).

Problem 4-1. Give asymptotic upper and lower bounds for \(T(n)\) in each of the following recurrences. Assume that \(T(n)\) is constant for \(n \leq 2\). Make sure your bounds are as tight as possible and justify your answers.

- (c) \(T(n) = 16T(n/4) + n^2\).
- (d) \(T(n) = 7T(n/3) + n^2\).
- (e) \(T(n) = 7T(n/2) + n^2\).
- (f) \(T(n) = 2T(n/4) + \sqrt{n}\).

Exercise 7.3-3. Show that there are at most \([n/2^{h+1}]\) nodes of height \(h\) in any \(n\)-element heap.

Exercise 8.3-3. Describe an implementation of \(\text{RANDOM}(a, b)\) that uses only fair coin flips. What is the expected running time of your procedure?