CA 765A/ECE 697: Real-Time Systems Homework 4
Off-Campus Students: Due within one week of watching Lecture 15

(1) Compute the reliability of an NMR system under the following conditions. Processors fail with probability $p$, and the (single) voter fails with probability $\sqrt{N}v$. For $p = 10^{-4}$ and $v = 10^{-5}$, plot the reliability of the system as a function of $N$, for $N$ varying from 3 to 27, in steps of 2.

(2) Note: This problem is very similar to that solved in pages 340 to 343 of the text: the main difference is that the execution time here is uniformly distributed, and the goal is to find the availability, not the reliability.

In this problem, you will study the impact of fault latency. You have a TMR system with a perfectly reliable voter. Only permanent faults happen: there are no transients. Processors fail independently according to a Poisson process with rate $\lambda$, and the fault latency is exponentially distributed with parameter $\mu$.

The system executes a single task in an endless loop: as soon as it finishes one iteration, it votes on the result and immediately starts the next iteration. Voting takes negligible time. It is only at the vote that error-generating processors can be identified. The execution time of each iteration is not constant: it is uniformly distributed over the interval $[a, b]$.

When an error is detected at the voter, the offending processor is replaced by a spare. The spare is guaranteed to be good at the moment it is installed. You have an infinite supply of spares. Replacement time is negligible.

System failure is only defined as happening at a voting instant: if two or more processors are generating errors at this time, the system fails.

Determine the availability of this system. That is, find the fraction of the iterations in which the system will be functional. Follow these steps:

- Define states to capture the relevant information about the system.
- Study the transitions over the execution time of an iteration.
- Given the state of the system at the beginning of an iteration, obtain the state probabilities at the end of the iteration. This is the probability transition matrix associated with the evolution of the system over the course of an execution.
- Given the state of the system at the end of the $i$th vote (after any processors detected as faulty have been replaced), find the state probabilities at the end of the $i + 1$st vote.
• From the above steps, determine the system failure probability per iteration.

With these equations developed, carry out the following numerical study of the impact of the various parameters.

- Fix $a = 5$, $b = 10$. Plot curves for the system availability as a function of $\mu$, where $\mu$ varies from 1 to 50: draw curves for each of the following values of $\lambda$: $10^{-3}$, $10^{-1}$, and $10^{-5}$.

- Discuss any interesting features in your numerical results.

- Attach your fully documented source code to the homework.