Selection of On-Line Measurements for Nonlinear Parameter Estimation

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Outline

- Motivation
- Existing measurement selection methods
- Proposed method
  - Basic characteristics
  - Iterative selection procedure
- Nitrogen purification column example
- Conclusion and future work

Motivation

- **Model-based control of distillation columns**
  - Simple nonlinear models suitable for on-line use
  - Simple models often not sufficiently accurate
  - Model adaptation may be necessary
- **On-line nonlinear parameter estimation**
  - Estimated parameters posed as unmeasured state variables
  - Nonlinear estimator designed for augmented system
  - Estimator provides updated state/parameter values
  - Many potential measurement locations (e.g., each equilibrium stage)
  - Systematic selection procedures needed

Motivation (cont.)

- Parameter selection problem
  - Given a set of measurements, determine the set of parameters that can be most effectively estimated
  - Sandink et al., Industrial & Engineering Chemistry Research (2001)
  - Li et al., IEEE Trans. on Control Sys. Technol. (accepted)
- Measurement selection problem
  - Given a set of estimated parameters, determine the set of measurements that provide the most useful information for estimation
  - Motivated by our work on nonlinear wave models for cryogenic air separation columns
  - Especially important for high-purity columns
  - General procedure proposed in this talk

Existing Measurement Selection Methods

- Engineering judgment
- Mathematical approaches
  - Based on steady state gain information

$$K_{ij} = \frac{\Delta y_i / y_i}{\Delta p_j / p_j}$$

p_j : Nominal value of the parameter

Δp_j: Imposed change in the parameter

y_i : Nominal value of the measured output

Δy_i : Observed change in the measured output

  - Singular Value Decomposition (SVD)

SVD-Based Measurement Selection Procedure

- Procedure (*Luyben, Practical Distillation Control,1992*)
  - Perform SVD on scaled gain matrix: $K=U\Sigma V^T$
  - Select the largest element of the first left singular vector as the first measurement location
  - Select the largest element of the second left singular vector as the second measurement location
  - Continue until the number of measurements equals the number of parameters
- Limitations
  - Only applicable when the number of selected measurements is less than or equal to the number of estimated parameters
  - Only a single steady-state operating point is considered

Proposed Method

- Objectives
  - Determine the best measurement locations
  - Overcome limitations of SVD analysis
- Basic idea
  - Use Principal Component Analysis (PCA) to measure sensitivity of candidate measurements
  - Use angle between sensitivity vectors as a measure of linear independence of candidate measurements
  - Use two measures to develop iterative measurement selection procedure

Overall Effect Measure

- Compute scaled gain matrix: $K \in R^{m \times p}$
- Calculate covariance matrix: $X = KK^T \in R^{m \times m}$
- Perform PCA on X
- Arrange Eigenvalues and Eigenvectors such that

$$\begin{array}{ccccccccccc} \lambda_1 \geq & \lambda_2 \geq & \cdots \geq & \lambda_p \geq & \lambda_{p+1} = & \cdots = & \lambda_m = 0 \\ C_1 & C_2 & & C_p & C_{p+1} & & C_m \end{array}$$

- Calculate overall effect of i-th measurement

$$E_i = \frac{\sum_{j=1}^{m} \left| \lambda_j C_{ji} \right|}{\sum_{j=1}^{m} \left| \lambda_j \right|}$$

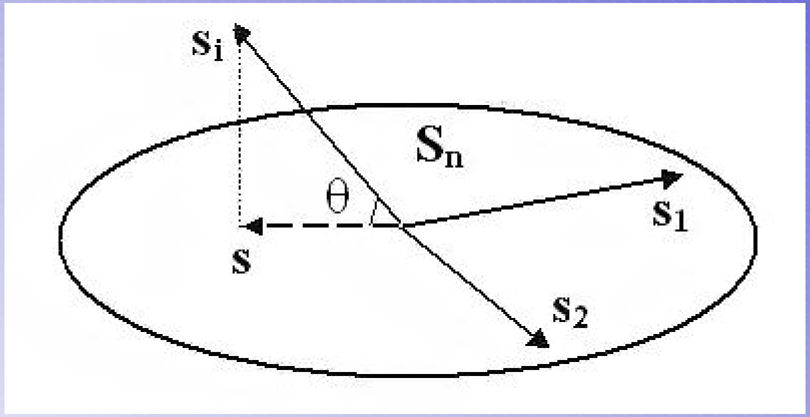
- Does not account for linear dependence between measurements

Linear Independence Measure

- Use the angle between sensitivity vectors as a measure of their linear independence
  - Vector 1: sensitivity vector of candidate measurement (s_i)
  - Vector 2: vector closest to s_i in the space (S_n) spanned by sensitivity vectors of measurements already chosen (s_1, s_2 , …)

$$\theta = \cos^{-1}\left(\frac{s_i^T s}{\|s_i\|\|s\|}\right)$$

$$d_i = \sin\theta$$



- $d_i=1$: measurement offers completely new information
- $d_i=0$: measurement is redundant

Measurement Selection Algorithm

- 1st : Select the measurement with largest E_i value
- 2nd -pth : Select the measurement with largest $E_i d_i$ value
- p+1th -mth : Form all possible (*p-1*)-tuples of the previously selected measurements.

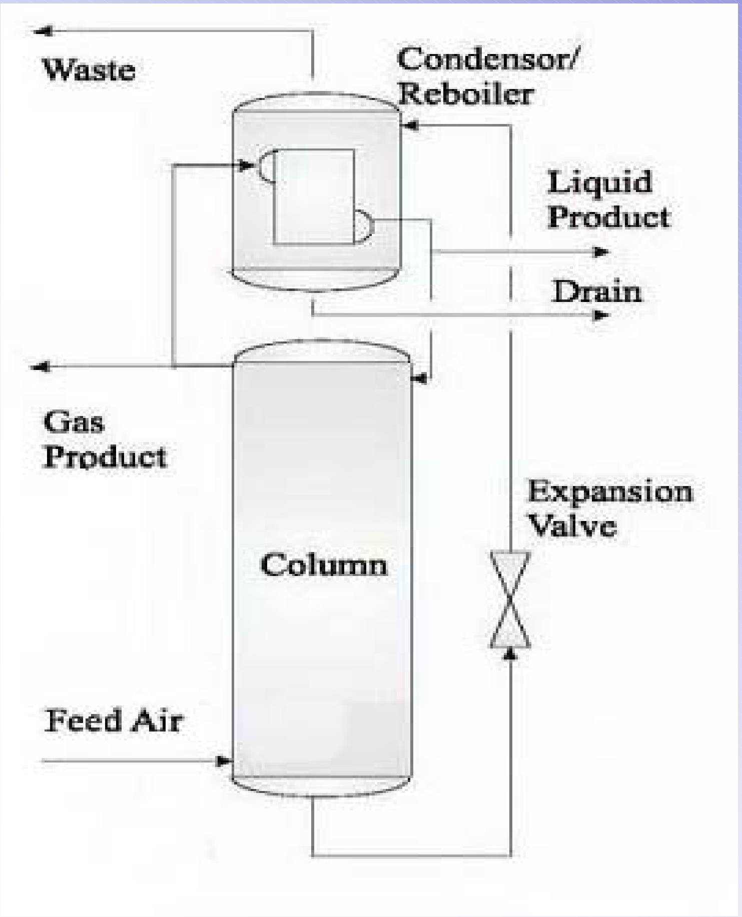
$$q = \frac{k!}{(p-1)!(k-p+1)!}, \quad k \in [p, m]$$

Worse case: $d_i = min_q \, d_{q,i}$

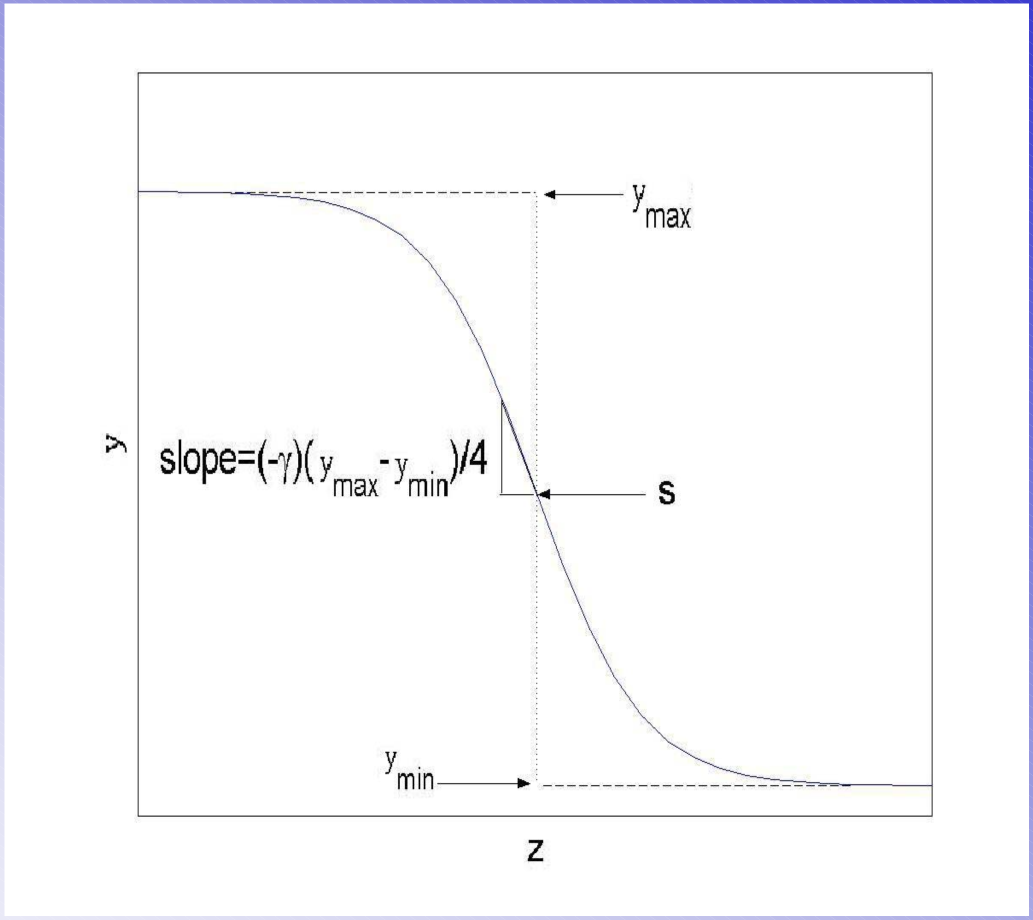
Select measurement with the largest $E_i d_i$ value

Example: Wave Model for a Rectifying Column

Nitrogen purification column



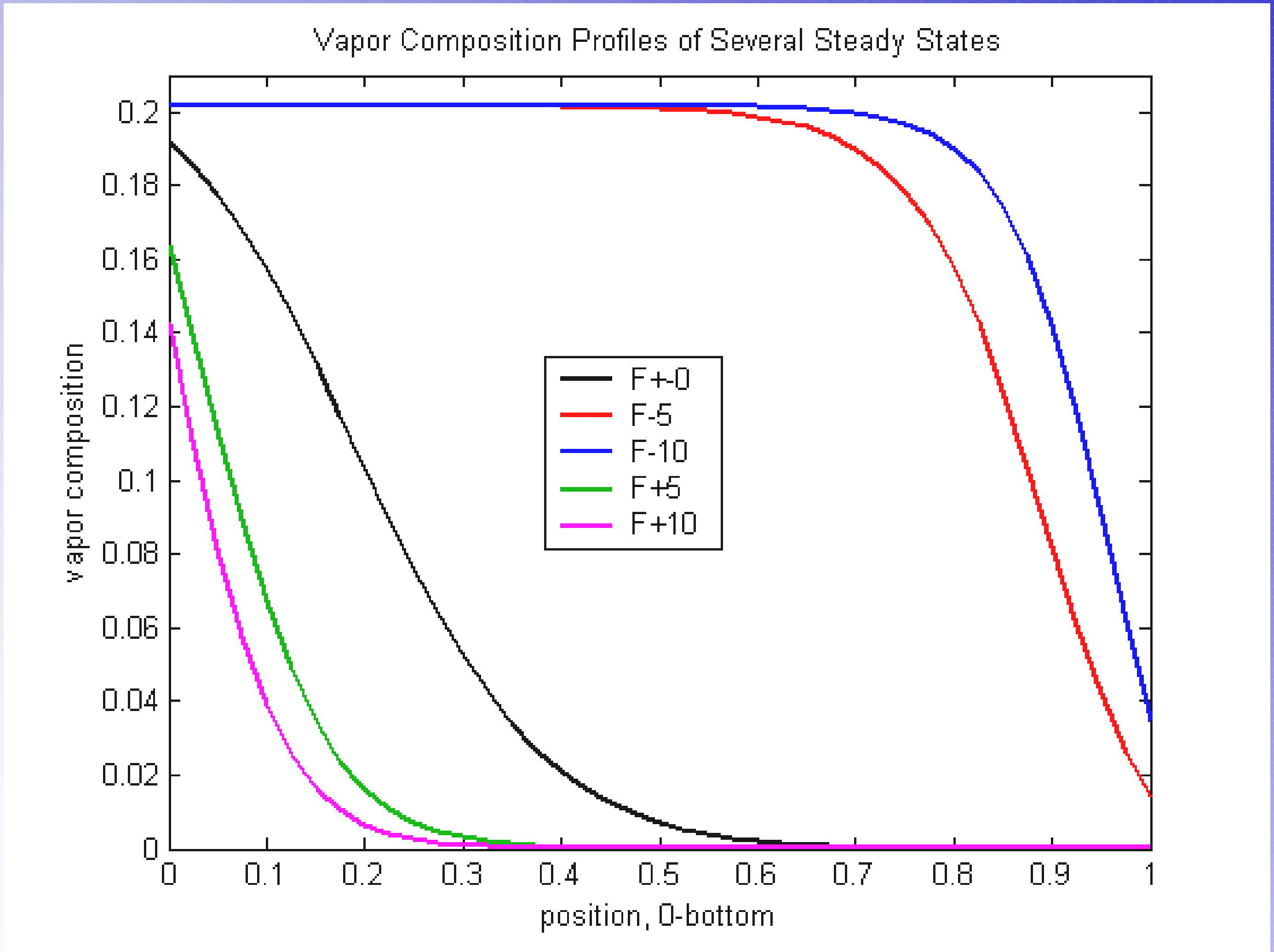
Nonlinear wave model



Nonlinear State and Parameter Estimation

- Estimate wave position (s) and wave front slope (γ) and upper limit parameter (y_{max})
- Online estimation problem
  - Objective is accurate reconstruction of composition profile
  - 3 estimated variables and 41 possible measurements
  - Measurements may be tray compositions or temperatures
  - Need one more measurement than the number of estimated parameters
  - On-line estimation performed with Extended Kalman Filter (EKF)
- Operating Conditions
  - 5 representative steady states considered
  - Nominal (F=98.4 kmol/hr), F±5, F±10
  - Data generated with Aspen simulator

5 Steady States



3 Estimated Variables

- SVD method: nominal steady state only

Measurement location sequence	1	2	3	4	5	6
41 log transformed liquid compositions	1	32	41	N/A	N/A	N/A
41 temperature measurements	32	37	41	N/A	N/A	N/A
Log transformed 1st tray liquid composition and 40 temperatures	1	33	37	N/A	N/A	N/A

- Proposed method: nominal steady state only

Measurement location sequence	1	2	3	4	5	6
41 log transformed liquid compositions	1	32	40	25	36	15
41 temperature measurements	32	37	41	27	39	23
Log transformed 1st tray liquid composition and 40 temperatures	1	33	28	37	23	39

Measurement Selection Results

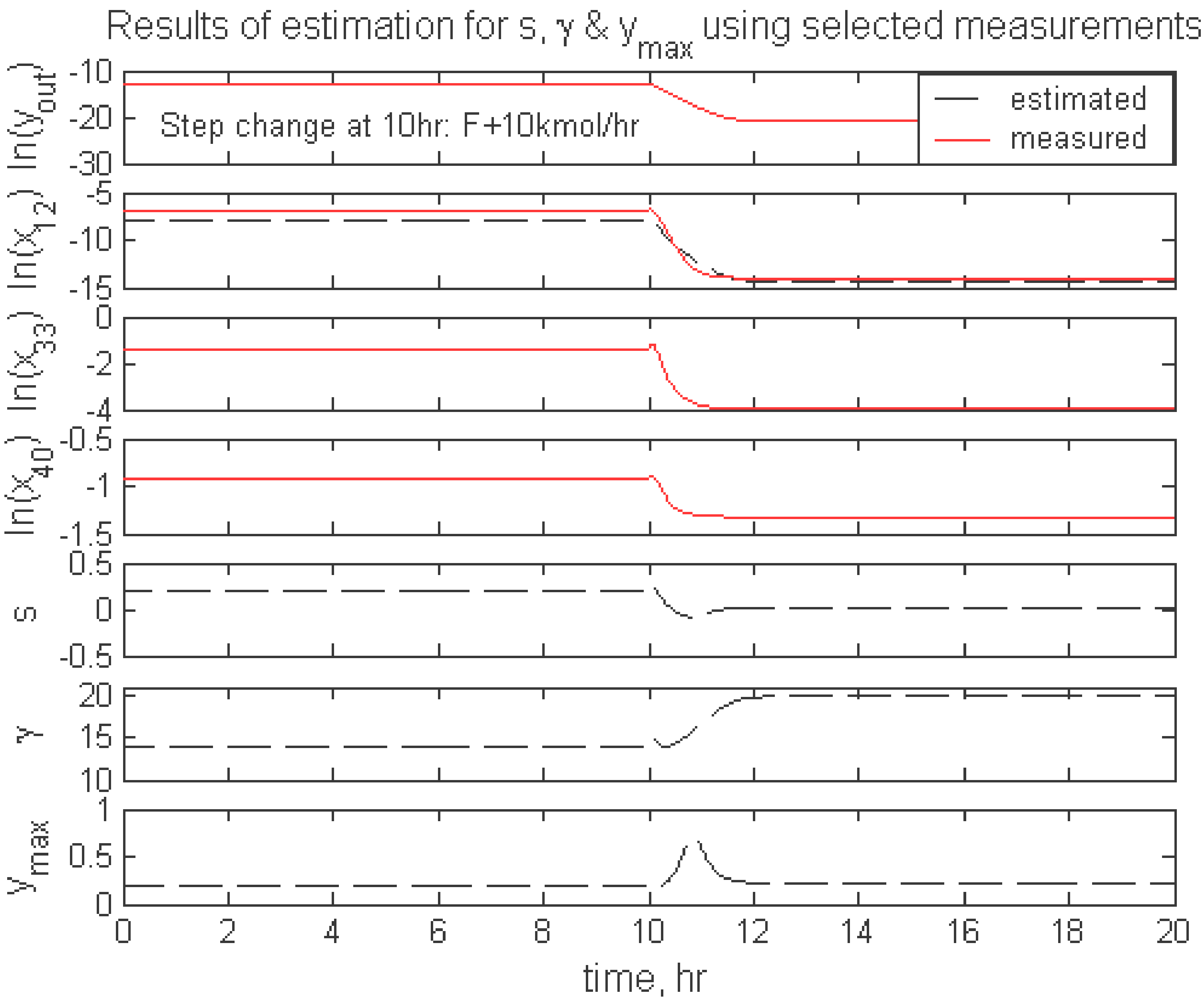
- Selection based on log transformed liquid compositions

Measurement location sequence	1	2	3	4	5	6
Nominal	1	32	40	25	36	15
F-5	3	9	1	6	11	41
F-10	1	7	4	2	5	8
F+5	33	1	40	37	21	39
F+10	30	38	40	1	21	35

- Analysis of averaged gain matrix

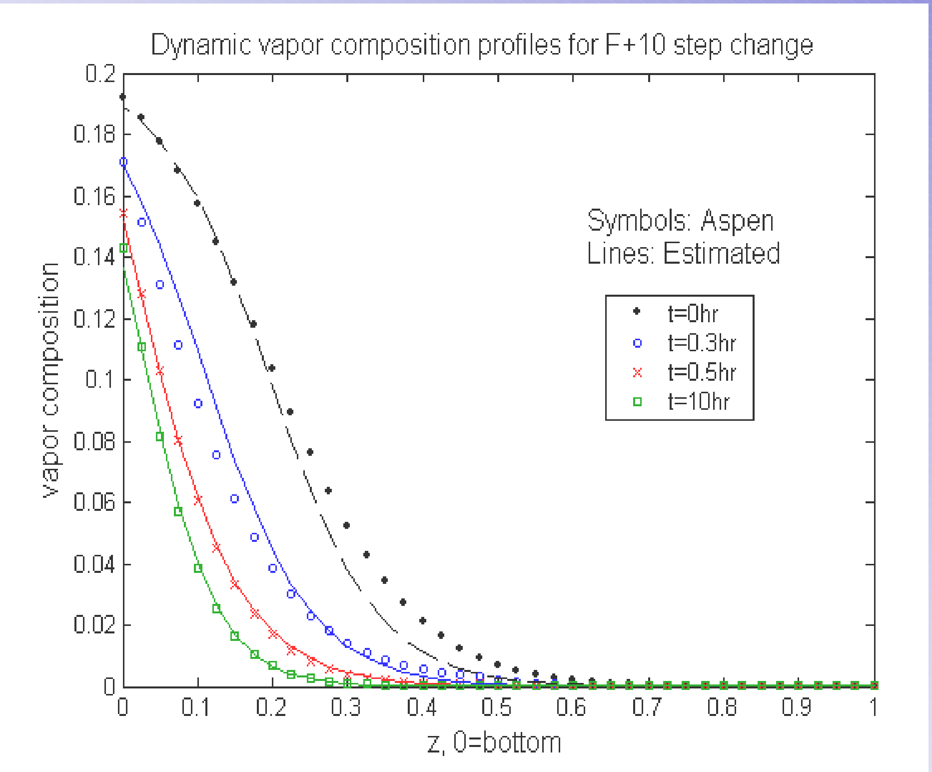
Measurement location sequence	1	2	3	4	5	6
Averaged gain matrix over 5 SS's	1	33	12	40	3	41

Online Estimation



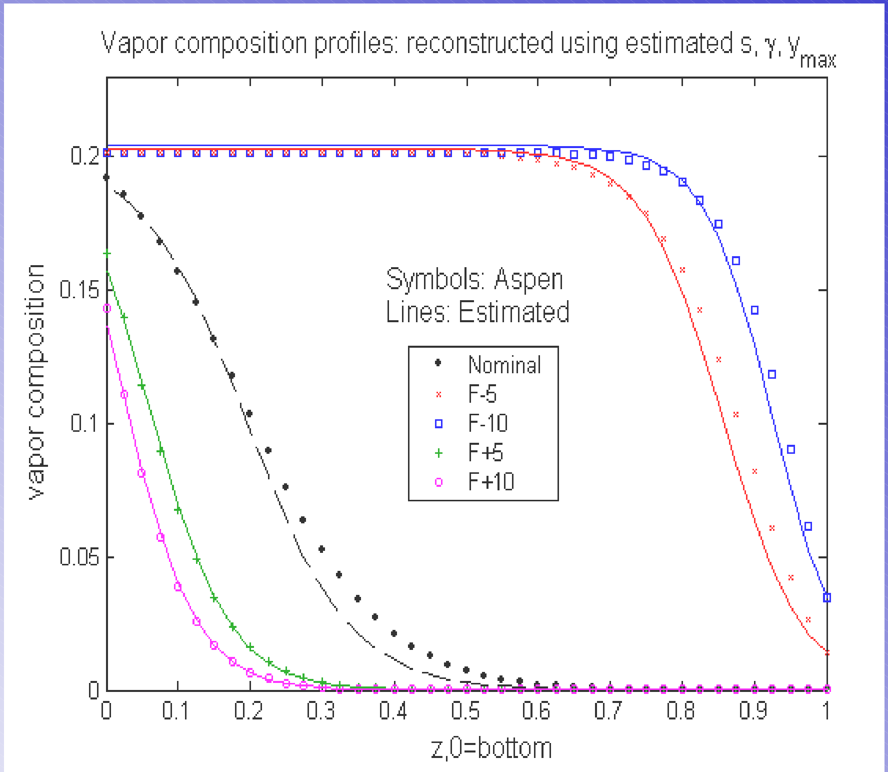
Reconstructed Profiles

Dynamic Profiles



Nominal SS to F+10 SS

Steady State Profiles



5 Steady States

Conclusions and Future Work

- New measurement selection method
  - Yields similar results to SVD when the number of selected measurements is equal to the number of estimated parameters
  - Allows the number of selected measurements to be greater than the number of estimated parameters
  - Applied successfully to a rectifying column
- Future work
  - Application of the selection method to a column with both rectifying and stripping sections