CA 770A/ECE 673 Test 1

75 minutes; Closed Book/Notes

Be sure to write your name on each page and show all work clearly.

Name: ______________________________

ID No: ______________

Question Number	Max Marks	Marks Subtracted
1	15	
2	20	
3a	10	
3b	10	
4	20	
5	20	
Total Subtracted		
Total Marks		

Name: ______________________

(1) [15 marks] Consider an M/M/1 queue, with arrival rate λ and service rate $\mu = \lambda/2$. Denote by t_i the time when job i departs the queue. Assume that job i leaves behind an empty system. What is the probability density function of the interval between the departures of job i and job $i+1$? That is, what is the pdf of $t_{i+1} - t_i$?

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(2) [20 marks] We have a queue whose server takes *vacations*. Whenever the queue becomes empty, the server goes away for a vacation, whose duration has probability density function, $\nu(t) = \alpha e^{-\alpha t}$. If the server comes back from a vacation to find the queue still empty, it goes away on another vacation. Vacation times are independent and identically distributed. Arrivals to this queue are according to a Possion process with rate λ. What is the probability that the server returns from a vacation to find the queue still empty?

Name:________________________

(3) [20 marks] Given a random number generator which produces uniformly distributed numbers between 0 and 1, show how you would generate a:

- (a) Discrete random variable, X, with probability mass function given by

$$P\{X = i\} = \frac{e^{-\lambda}\lambda^i/i!}{\sum_{j=0}^{k} e^{-\lambda}\lambda^j/j!}, \qquad i = 0, \cdots, k$$

Name:____________________

- (b) Continuous random variable, $Z = Y_1 + Y_2$, where Y_1 is exponentially distributed with parameter μ and Y_2 is uniformly distributed over the interval $[0, 1]$. Y_1 and Y_2 are independent.

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(4) [20 marks] You have a simulation program which outputs a random variable X. You run it till you collect 500 samples of X. 100 of these outputs are $X = 10$, 150 of them are $X = 12$, and the remaining 250 outputs are $X = 20$. Find the 80% and 95% confidence intervals associated with $E[X]$. Use the attached $N(0, 1)$ distribution and show all the steps in your computation. What would be the 100% confidence interval?

Name:________________________

(5) [20 marks] You have a coin, whose properties are initially unknown (i.e., our best estimate to begin with is that the probability of a head, p_H, is uniformly distributed between 0 and 1). You toss it a hundred times and get a head each time. What is the posterior density of the value of p_H after the information gained from these hundred tosses?

Name: ______________________

Blank Sheet: Use if you require additional space.