ECE 673: Homework 5

Due: April 20, 2006.

(1) Simulate a 3-stage butterfly network. Assume that p_i is the probability, for each input line, that a *new* request will be to set up a connection to the network output line i. Requests are repeated every cycle until they are satisfied. Each circuit is held for exactly one cycle. The goal is to obtain the expected bandwidth of this system.

Write a program, implementing the approximate bandwidth calculation we derived in class. In this approximation, unsatisfied requests are simply discarded: they have no impact on requests in a subsequent cycle.

Run your simulation model for $20,000$ cycles for each of the following cases. In each case, find the 95% confidence interval and compare it with the approximation you get from the analytical model.

(a) $p_i = 1/8$ for $i \in \{0, 1, \cdots 7\}$.

(b) $p_0 = 0.5$; $p_i = 0.05$ for $i \in \{1, 2, \cdots, 7\}$.

(2) Consider a crossbar switch. A crossbar can be thought of as a set of $n \times m$ switches, which permits any input-output permutation be satisfied, subject only to the requirement that the network output can support at most one circuit. If $p_{i,j}$ is the probability that input i wants to connect to output j, derive an expression for the expected bandwidth of the switch.

(3) Draw the Markov chain associated with an $M/M/\infty$ queue, where the arrival rate is λ and service parameter is μ per server. Write the differential equations associated with $\pi_i(t)$, the probability of being in state i at time t. (There is no need to try to solve these equations).

(4) By modifying the above Markov chain slightly, show how you can write equations to find the density function of the busy period of such a queue.

(5) Consider driving down North Pleasant Street on campus, which has a large number of pedestrian crosswalks. Assume that people arrive at the crosswalk according to a Poisson process with rate λ. They take an exponential amount of time, with parameter μ, to cross the street. You have to stop at the crosswalk until there is no pedestrian in it. Suppose you have to go through a total of 5 crosswalks.

The crosswalk can be modeled as an $M/M/\infty$ queue (it is assumed to have an infinite capacity for pedestrians). Suppose the busy period of such a queue has mean θ and second moment ϕ.

(a) What is the probability that you will not need to stop at any of the five crosswalks? Assume that the crosswalks are independent of one another.

(b) What is the total expected time you will spend waiting at the crosswalks?