# ECE 673: Homework 4

Due: April 11, 2006.

(1) Question 5.13 parts (a), (b) of Kleinrock (page 236).

(2) Question 5.19 of Kleinrock (page 238).

(3) Question 5.20 of Kleinrock (page 238).

(4) Question 5.22 parts (a), (b) of Kleinrock (page 239).

(5) Consider a token-ring network which uses exhaustive service. The token stays at a node, serving packets, until there are no further packets left in that node, and then moves on to the next node. Derive an implicit equation that is satisfied by $Q_i(z)$, the generating function of the probability that there are $n$ jobs waiting at node $i$ when the token gets there. (We have seen implicit equations before: recall the derivation of the Laplace transform of the busy period in M/G/1). Don't attempt to solve this equation. The purpose of this question is to give you some practice in the use of generating functions; this equation by itself is not of much other importance.

You can make the approximation that the times spent at each node in a cycle are mutually independent. Assume that the token-passing time from one node to the next is constant at $t_p$ seconds. Also, assume that the packets all have the same size, and serving a packet takes a fixed (not random) amount of time, $t_s$. Arrivals to each node are Poisson with rate $\lambda$.

(6) This question relates to Buzen's algorithm for closed queueing systems. Consider a closed queueing system with six servers. Each server has an exponentially distributed service time: the service rate is $\mu_i$ for server $i$. The routing probability from one server to the others is given by the following matrix:

$$R = \begin{pmatrix} 0.1 & 0.1 & 0.2 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.3 & 0.3 \\ 0.1 & 0.9 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix} \tag{1}$$

The service rates are: 1, 4, 5, 8, 1, 3 respectively. Assume there are 12 jobs in all. Using the convolution algorithm, compute the probabilities $P_i(n_i)$ of $n_i$ jobs in server $i$ for $i = 1, \cdots, 6$, and $n_i = 0, 1, \cdots, 12$. Hand in your program for implementing Buzen's algorithm, with suitable documentation to enable me to understand it.