ECE 673: Homework 2

Due: March 9 (On-Campus Students);
One week after watching Lecture 9 (Off-campus students).

(1) Consider a queueing system in which jobs always arrive in pairs. The pairs arrive according to a Poisson process with rate λ. Service is FCFS, with service time being exponentially distributed with parameter μ per job. (That is, while jobs arrive in pairs, they are served individually).

- (a) Draw the Markov chain for this queue and write the balance equations.
- (b) Obtain $\Pi(z) = \sum_{i=0}^{\infty} \pi_i z^i$ for this system, assuming steady state exists.
- (c) Use the generating function obtained in (b) above to find expressions for $\pi_0, \pi_1, \pi_2, \pi_3$. From these expressions, guess the general expression for π_n for any n. Then, check that your guess is correct by verifying that your expressions for π_n satisfy the balance equations.

(2) Prove that the union of independent Poisson processes with rates λ_1 and λ_2 is itself a Poisson process with rate $\lambda_1 + \lambda_2$. You can use the definition of Poisson processes provided in Ross (Chapter 2) or in Kleinrock (also Chapter 2).

(3) Consider a queue in which customers arrive according to a Poisson process with rate λ and have exponentially distributed service time, with parameter μ. The queue has one server. The customers exhibit impatience: if a customer is in position i of the queue, he leaves without waiting any further, with a probability of $i\gamma\Delta t + o(\Delta t)$ over an interval of time Δt.

Draw the Markov chain for this queue and write the balance equations for the steady-state probabilities.

(4) You have a five-processor system. Failure occur as Poisson processes with rate λ per processor. You have one repairman, who can work on one processor at a time: it takes an exponentially distributed amount of time, with mean $1/\mu$ to repair one processor. Find the steady-state probability that there will be n functional processors in the system.