ECE 655 Homework 4 Solutions

Q3 (Chapter 6). The execution time between checkpoints is $t_e = \frac{T}{N+1} + T_{ov}$. If there is a failure just before a checkpoint is completed, we have lost t_e of work. So, the worst-case execution time is

$$T_{wc} = kt_e + Nt_e$$

Differentiating with respect to N yields the optimum number of checkpoints:

$$N_{opt} = \sqrt{\frac{kT}{T_{ov}}} - 1$$

Q5a (Chapter 6). The slack time is $t_s = D - (T + NT_{ov})$. Define $t_1 = \frac{T}{N+1} + T_{ov}$. If $t_s \geq t_1$, a single point failure cannot cause a deadline miss, no matter when it occurs. On the other hand, if $t_s < t_1$, a deadline miss will occur if more than t_s of work is lost: the probability of this happening, given that exactly one failure has occurred is

$$\frac{t_1 - t_s}{t_1}$$

For convenience, denote by p_i the probability of the execution being hit by a total of i failures. The probability of a deadline miss is

$$p_{miss} = \sum_{i=1}^{\infty} p_i P(\text{Deadline miss if } i \text{ failures} | i \text{ failures})$$

A lower bound is obtained by using the inequality:

$$P(\text{Deadline miss if } i \text{ failures} | i \text{ failures}) \geq \text{Prob}\{\text{Deadline miss if 1 failure} | \text{1 failure}\}$$

for $i > 1$, and an upper bound by

$$\text{Prob}\{\text{Deadline miss if } i \text{ failures} | i \text{ failures}\} \leq 1$$

Thus, a lower bound of the deadline miss probability will be:

$$(1 - p_0)\frac{t_1 - t_s}{t_1}$$

An upper bound will be:

$$p_1 \frac{t_1 - t_s}{t_1} + 1 - (p_0 - p_1)$$

It only remains to calculate p_i, $i = 0, 1$. From the laws of a Poisson process, we can conclude that $p_0 = e^{-\lambda t_e}$. Now, consider the probability of exactly one failure over the entire execution. This is the probability that a failure happens sometime during the execution and that no subsequent failure occurs. Since the execution is broken down into a number of inter-checkpoint segments and the final segment, we can simply calculate the probability that one of these segments is hit by exactly one fault and that the others are not hit at all.

The probability, $h(\tau)$, that a segment of length τ is hit by exactly one fault can be calculated as follows:

$$h(\tau) = \int_{t=0}^{\tau} \text{Prob}\{\text{Segment hit for 1st time in } [t, t+dt]\}\text{Prob}\{\text{No subsequent failures beyond } t\}$$

$$= \int_{t=0}^{\tau} \text{Prob}\{\text{Segment not hit before } t\}\text{Prob}\{\text{Segment hit in } [t, t+dt]\}$$

$$\times \text{Prob}\{\text{No subsequent failures}\}$$

$$= \int_{t=0}^{\tau} e^{-\lambda t}\lambda e^{-\lambda \tau} dt$$

$$= e^{-\lambda\tau}\left(1 - e^{-\lambda\tau}\right)$$

Now, the execution is broken down into $N + 1$ segments. N of these segments are of length $\tau_1 = \frac{T}{N+1} + T_{ov}$ and the last is of length $\tau_2 = \frac{T}{N+1}$. From simple probability, we have

$$p_1 = \binom{N}{1} h(\tau_1)(1 - h(\tau_1))^{N-1}(1 - h(\tau_2)) + (1 - h(\tau_1))^N h(\tau_2)$$

Q7 (Chapter 6). Denote the checkpoints of the top process by $CP_1, CP_2, CP_3, CP_4, CP_5$ and the the checkpoints of the bottom process by CQ_1, CQ_2, CQ_3, CQ_4. The consistent recovery lines are: $\{CP_1, CQ_1\}$, $\{CP_1, CQ_2\}$, $\{CP_1, CQ_3\}$, $\{CP_2, CQ_2\}$, $\{CP_2, CQ_3\}$, $\{CP_3, CQ_2\}$, $\{CP_3, CQ_3\}$,

$\{CP_4, CQ_3\}$ and $\{CP_5, CQ_4\}$.

Q2 (Chapter 5). First, note that if $\pi_{bad} \le \pi_{stop}$, we obviously must set $\alpha = L$. So, let us consider the case $\pi_{bad} > \pi_{stop}$. The probability of a bad output falling inside the acceptable interval is $\frac{\alpha}{L}$. The probability of a good output falling outside the acceptable interval is

$$\int_{\alpha}^{L} \frac{\mu e^{-\mu x}}{1 - e^{-\mu L}} dx = \frac{e^{-\mu \alpha} - e^{-\mu L}}{1 - e^{-\mu L}}$$

Given that q is the probability of producing a bad output, we have the expected cost as

$$C(\alpha) = \pi_{bad}\ q\frac{\alpha}{L} + \pi_{stop}\left(q\left(1 - \frac{\alpha}{L}\right) + (1-q)\frac{e^{-\mu\alpha} - e^{-\mu L}}{1 - e^{-\mu L}}\right)$$

So, find α such that $C'(\alpha) = 0$: this can be shown, by calculus, to be given by

$$\alpha = -\frac{1}{\mu} \ln\left(\frac{q(\pi_{bad} - \pi_{stop})(1 - e^{-\mu L})}{\mu \pi_{stop}(1-q)L}\right)$$

It is easy to check that $C''(\alpha) > 0$, thus meeting the requirement of a minimum.

Q4a (Chapter 5). We want to calculate $\text{Prob}\{A|B\}$. Based on the conditional probability formula, we have:

$$\begin{aligned} \text{Prob}\{A|B\} &= \frac{\text{Prob}\{A \cap B\}}{\text{Prob}(\{B\}} \\ &= \frac{\text{Prob}\{B|A\}\text{Prob}\{A\}}{\text{Prob}(\{B \cap A\} + \text{Prob}(\{B \cap C\}} \end{aligned}$$

Let us look at each of the terms in this expression. Clearly, $\text{Prob}\{B|A\} = 1$. As for $\text{Prob}\{A\}$, the best estimate we have is that this is equal to p. To calculate $\text{Prob}\{B \cap A\}$ and $\text{Prob}\{B \cap C\}$, we will need to invoke the conditional probability formula once more:

$$\begin{aligned} \text{Prob}\{B \cap A\} &= \text{Prob}\{B|A\}\text{Prob}\{A\} \\ &= p \\ \text{Prob}\{B \cap C\} &= \text{Prob}\{B|C\}\text{Prob}\{C\} \\ &= e^{-\mu t} q \end{aligned}$$

Once again, we are using the best estimates we have for $\text{Prob}\{A\}$ and $\text{Prob}\{B\}$. Substituting back into the expression for $\text{Prob}\{A|B\}$, we have:

$$\text{Prob}\{A|B\} = \frac{p}{p + qe^{-\mu t}}$$