Adaptive-Modulation Schemes for Minimum Outage Probability in Wireless Systems

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***Abstract*—Adaptive-modulation schemes are designed that yield the minimum outage probability for wireless systems with strict delay constraints, under the assumption of perfect causal channel state information at the transmitter and receiver. Numerical results indicate that the proposed schemes significantly outperform adaptive schemes designed to maximize the average rate.**

***Index Terms*—Adaptive modulation, fading channels, outage probability.**

I. Introduction

PREVIOUS adaptive-modulation schemes [1]–[3] have attempted to maximize the average data rate subject to a bit-error rate (BER) constraint, and thus, apply primarily to systems with no delay constraints. However, most of today's mobile radio systems carry real-time speech, while streaming audio and video are predicted to be prominent applications of the next-generation Internet. These applications impose delay constraints, and thus *outage probability*, defined here as the probability of making a bit error or not being able to meet the delay constraint, is an appropriate performance indicator for the communication system.

In this letter, a discrete-rate adaptive-modulation scheme with constant transmit power is designed for minimum outage probability under a strict delay constraint. Accurate causal channel state information (CSI) at the transmitter and receiver is assumed. Motivated by the optimal scheme, a simple suboptimal algorithm is also proposed. The proposed schemes are compared with adaptive schemes designed to maximize the average rate, subject to a bit-error probability (BEP) constraint [1].

The remainder of this paper is organized as follows. The system model is presented in Section II. In Section III, the optimal design of a discrete-rate adaptive scheme is outlined, which is followed by the presentation of the suboptimal scheme in Section IV. Finally, numerical results and conclusions are presented in Section V.

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II. System Model

A baseband system model for the problem is shown in Fig. 1 and follows that in [1]. The independent and identically distributed (i.i.d.) sequence $b[k]$ is the stream of information bits to be transmitted over the channel. The discrete-time channel is modeled as a zero-mean stationary complex Gaussian random process $\{X_k\}$ with independent real and imaginary parts $\{X_{R,k}\}$ and $\{X_{I,k}\}$, respectively, each with autocorrelation function $R_X[k] = E[X_{R,l}X_{R,l+k}] = E[X_{I,l}X_{I,l+k}]$. Adaptive communication concerns choosing transmitter parameters for the $(k+1)$th symbol, given $X_l = x_l$, $l = 1,\cdots,k$, where x_l is the value that the random variable X_l assumed. For any sequence $u_0, u_1, \ldots, u_k$, define the notation $\underline{u}_{k:1} = (u_k, u_{k-1}, \ldots, u_1)$. Under the assumption of a known channel autocorrelation function, $R_X[k]$, the channel gain $Y = |X_{k+1}|$ for the $(k+1)$th symbol interval, conditioned on $\underline{X}_{k:1}$, is Rician and has a probability density function given by [3]

$$p_{Y|\underline{X}_{k:1}}(y|\underline{x}_{k:1}) = \frac{y}{\sigma^2} e^{-\frac{y^2+s^2}{2\sigma^2}} I_0\left(\frac{ys}{\sigma^2}\right) \quad y \geq 0 \tag{1}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function, $s^2 = (\underline{\rho}^T \Sigma_{\underline{X}_{k:1}}^{-1} \underline{x}_{R,k:1})^2 + (\underline{\rho}^T \Sigma_{\underline{X}_{k:1}}^{-1} \underline{x}_{I,k:1})^2$ is the squared noncentrality parameter, $\Sigma_{\underline{X}_{k:1}} = E[\underline{X}_{R,k:1}\underline{X}_{R,k:1}^T] = E[\underline{X}_{I,k:1}\underline{X}_{I,k:1}^T]$ is the k by k autocorrelation matrix of the real or imaginary component of the outdated estimates, and $\underline{\rho} = E[\underline{X}_{R,k:1}X_{R,k+1}] = E[\underline{X}_I X_{I,k+1}]$ is the k-dimensional column correlation vector. The parameter σ^2 is the mean squared error (MSE) of a minimum MSE (MMSE) predictor of the in-phase (or quadrature) fading of interest, and is given by $\sigma^2 = R_X[0] - \underline{\rho}^T \Sigma_{\underline{X}_{k:1}}^{-1} \underline{\rho}$.

For a delay-constrained application, the higher layers shall provide the communication system with the total number of information bits N to be transmitted, and the delay constraint K in symbol intervals. The variables τ_e and τ_f represent the delays involved in obtaining a channel estimate at the receiver, and of providing such to the transmitter, respectively. However, here it will be assumed that coherent reception with perfect carrier-phase estimation and perfect fading-value estimation $(\epsilon = 0)$ at the receiver $(\tau_e = 0)$ and the transmitter $(\tau_f = 0)$ during data transmission is assumed in this letter. However, it is noted that the $\tau_e \neq 0$, $\tau_f \neq 0$, and $\epsilon \neq 0$ cases are conceptually similar [3]. A constant-power variable-rate adaptive quadrature amplitude modulation (QAM) system is considered with average received signal-to-noise ratio (SNR) E_s/N_o, where E_s is the average received energy per QAM symbol, and $n[k]$ is an i.i.d. sequence of zero-mean complex Gaussian random variables, each with real and imaginary components with variance $N_o/2$.

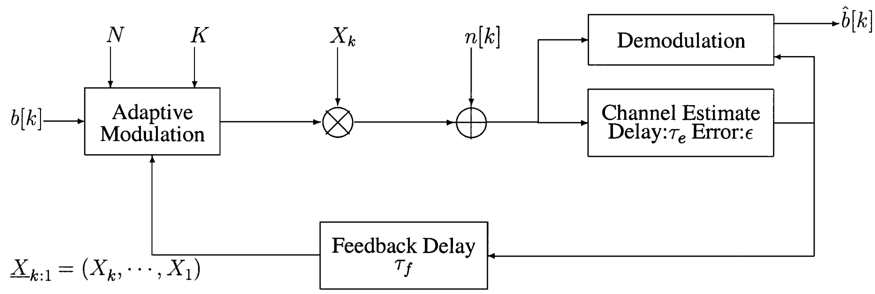

Fig. 1. System model.

III. Optimal Adaptive Scheme

The goal of an adaptive-signaling scheme is to use channel knowledge at the transmitter to optimize the transmitted waveform. Here, an algorithm for determining the optimal number of information bits to transmit on the current symbol is derived. The outage probability is defined as

$$\begin{aligned}\Pr(\text{outage}) = Pr\,(&\{\text{Error in any of the } K \text{ symbol intervals}\}\\ &\cup\{\text{Less than } N \text{ bits sent in } K \text{ symbol intervals}\}).\end{aligned}$$

Unlike traditional adaptive-signaling techniques, the adaptation scheme optimized for outage will always send N bits in K symbol intervals (sending all remaining bits in the last symbol interval if required), and thus, the outage probability is equal to the probability of at least one symbol error in the K symbol intervals.

Let N_i be the number of bits sent in the ith symbol interval, and N_k^+ represent the number of bits remaining to be sent at the kth symbol interval, i.e., $N_k^+ = N - \sum_{i=1}^{k-1} N_i$. Also, let $P_{ei}(x_i, N_i)$ represent the symbol-error probability (SEP) in the ith symbol interval conditioned on the current channel fading x_i and the number of information bits N_i sent, $i = 1, 2, \ldots, K$. The outage probability, $P_{oi}(\underline{x}_{i:1}, N_i^+, N_i)$, defined as the probability of making an error in the current *or* any of the subsequent symbol intervals, given N_i bits are sent in the current symbol interval and the remaining $N_i^+ - N_i$ bits are sent in the subsequent $K - i$ symbol intervals, is given as

$$\begin{aligned}P_{oi}\left(\underline{x}_{i:1}, N_i^+, N_i\right) &= P_{ei}(x_i, N_i)\\ &+ (1 - P_{ei}(x_i, N_i))\,\overline{P}_{o(i+1)}\left(\underline{x}_{i:1}, N_i^+ - N_i\right). \quad (2)\end{aligned}$$

The term $\overline{P}_{o(i+1)}$ represents the probability of making an error in any of the subsequent intervals, and will be defined precisely below.

Using the above notation, the minimum outage probability for the ith symbol interval when N_i^+ bits need to be sent in the remaining $K - i + 1$ symbol intervals, $P_{oi}^*(\underline{x}_{i:1}, N_i^+)$, is given by

$$P_{oi}^*\left(\underline{x}_{i:1}, N_i^+\right) = \min_{0 \le N_i \le N_i^+} P_{oi}\left(\underline{x}_{i:1}, N_i^+, N_i\right). \quad (3)$$

Equation (3) requires a computation of the outage probability (2). Solution of (2), in turn, depends on $\overline{P}_{o(i+1)}$, the average probability of an error in any of the subsequent intervals, given the channel-fading history, $\underline{x}_{i:1}$. Using (1), the conditional distribution of the channel fading for the subsequent symbol interval $p(x_{i+1}|\underline{x}_{i:1})$ can be obtained. Thus, by averaging the minimum outage probabilities of the subsequent intervals over the conditioned channel-fading density function

$$\begin{aligned}&\overline{P}_{o(i+1)}\left(\underline{x}_{i:1}, N_i^+ - N_i\right)\\ &\quad = \int P_{o(i+1)}^*\left(\underline{x}_{i+1:1}, N_i^+ - N_i\right) p(x_{i+1}|\underline{x}_{i:1})dx_{i+1} \quad (4)\end{aligned}$$

with the boundary condition being

$$P_{oK}^*\left(\underline{x}_{K:1}, N_K^+\right) = P_{ei}\left(x_K, N_K^+\right). \quad (5)$$

For the case of N bits being transmitted in K symbol intervals and assuming H discrete channel-fading values, the problem of choosing the optimal signal set size has a $O(H^{K-1}N^2)$ complexity. Hence, the optimal solution is intractable. However, if the channel can be modeled as first-order Markovian [4], which is true for slowly fading channels [5], [6], the complexity reduces to $O(HKN^2)$. The optimal adaptive-modulation scheme given by (2)–(5) then reduces to

$$\begin{aligned}&P_{oi}\left(x_i, N_i^+, N_i\right)\\ &\quad = P_{ei}(x_i, N_i) + (1 - P_{ei}(x_i, N_i))\,\overline{P}_{o(i+1)}\left(x_i, N_i^+ - N_i\right) \quad (6)\end{aligned}$$

$$P_{oi}^*\left(x_i, N_i^+\right) = \min_{0 \le N_i \le N_i^+} P_{oi}\left(x_i, N_i^+, N_i\right) \quad (7)$$

$$\begin{aligned}&\overline{P}_{o(i+1)}\left(x_i, N_i^+ - N_i\right)\\ &\quad = \int P_{o(i+1)}^*\left(x_{i+1}, N_i^+ - N_i\right) p(x_{i+1}|x_i)dx_{i+1} \quad (8)\end{aligned}$$

while the boundary condition remains unchanged.

Representing the magnitude of the channel fade for the ith symbol by $h_i = |x_i|$, the optimal adaptive scheme can be specified by a table of thresholds, $h_p(n, k)$, $p = 0, 1, \ldots, \log_2 M$; $n = 0, \ldots, N$; $k = 1, \ldots, K$, where M is the maximum number of signal points in a constellation. The optimal adaptive scheme will employ 2^p-QAM if n bits need to be transmitted in k symbol intervals and $h_p(n, k) \le h_i < h_{p+1}(n, k)$.

IV. Suboptimal Scheme

The optimal scheme generates $(N + 1) \cdot K \cdot (\log_2 M + 1)$ thresholds, $h_p(n, k)$. With the goal of reducing memory requirements, the suboptimal algorithm will start with a coarse sampling in time (or, equivalently, in symbol number) of these thresholds, given by $sh_p(j), j = 1, \ldots, K/k_s$, where the coarseness of the sampling is determined by its input parameter k_s. These thresholds are then tuned by increasing or decreasing them depending on how far "ahead" or "behind" the system is

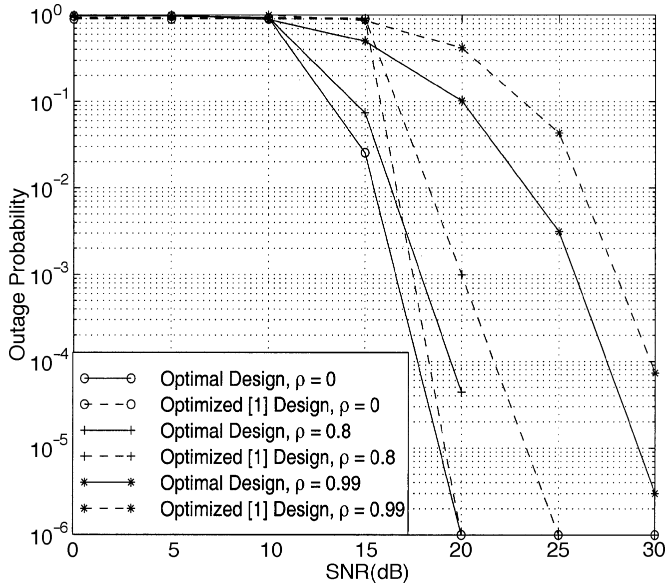

Fig. 2. Outage probability versus SNR under a delay constraint requiring 200 bits to be sent in 100 symbol intervals. Solid lines represent the performance of the optimal design, and dashed lines represent that of [1] for different values of channel correlation $\rho = R_X[1]$ between a component (real or imaginary) of the fading on adjacent symbols.

in sending the information bits, to get the final set of thresholds $th_p(j)$. The tuning parameter δ_h is obtained via simulation.

Suboptimal $(\mathbf{N}, \mathbf{K}, \mathbf{M}, k_s, \delta_h)$
$n = N$
$sh_p(j) \leftarrow h_p((N/K)jk_s, jk_s)$; $j = 1, \ldots, K/k_s$, $p = 0, \ldots, \log_2 M$
do $k = K : 1$
  do $p = 0 : \log_2 M$
    $th_p \leftarrow sh_p(\lceil k/k_s \rceil) + ((N/K)k - n) \times \delta_h$
  end
  $n \leftarrow n - p\{th_p \leq |x_k| < th_{p+1}\}$
end

V. Numerical Results and Conclusions

Simulation results for the outage probability of various schemes are shown in Figs. 2–4. In each case, the channel is assumed to be first-order Markovian with correlation coefficient ρ of the real (or imaginary) portions of adjacent fading values. Thus, channels with a small ρ, where adjacent fading values exhibit a low degree of correlation, will tend to exhibit a large amount of fading variation over a packet duration. Likewise, channels with a large ρ, where adjacent fading values exhibit a high degree of correlation, will tend to exhibit a small amount of fading variation over a packet duration.

Fig. 2 investigates relative performance gains as a function of ρ for various adaptive algorithms. Observe that for a channel exhibiting fading with low correlation in time ($\rho = 0$), the optimal design achieves a 1-dB performance gain over the scheme of [1] at an outage probability of 10^{-3}. However, for the same delay constraint, the gain increases to 2.25 dB when a channel exhibiting fading with high correlation in time ($\rho = 0.99$) is considered. Fig. 3 investigates the impact of the delay constraint on the gains observed. For the looser data constraint requiring 200 bits to be sent in 100 symbol intervals, the optimal design has a 2.25-dB gain over that of [1] at an outage probability of 10^{-3}. However, under the tighter delay constraint that requires 20 bits to be sent in 10 symbol intervals, the gain increases to 3.25 dB.

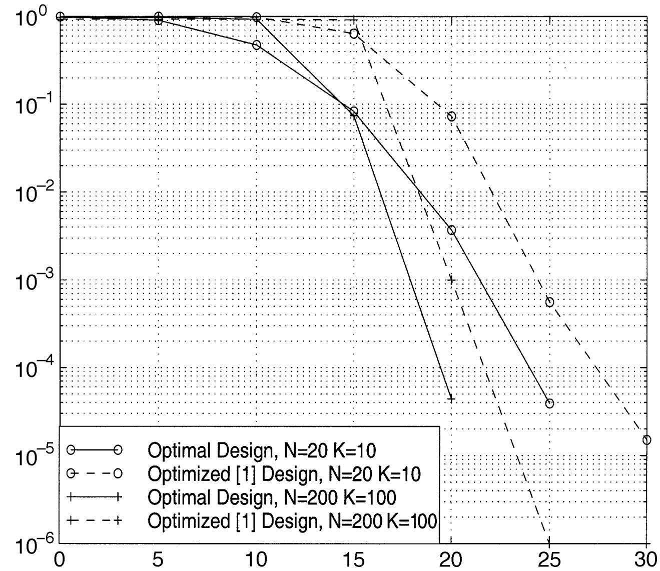

Fig. 3. Outage probability versus SNR for a correlation $\rho = R_X[1] = 0.8$ between a component (real or imaginary) of the fading on adjacent symbols. Solid lines represent the performance of the optimal design, and dashed lines represent that of [1] for different delay constraints.

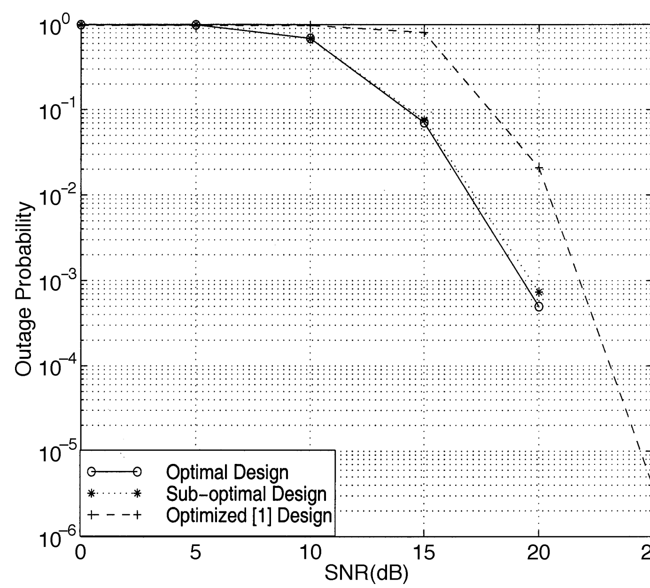

Fig. 4. Outage probability versus SNR for a delay constraint requiring 60 bits to be sent in 30 symbols for a correlation $\rho = R_X[1] = 0.8$ between a component (real or imaginary) of the fading on adjacent symbols. Solid line represents the performance of the optimal design, dashed line represents the performance of [1], and dotted line represents that of the suboptimal scheme.

Thus, in general, the performance gain of the proposed schemes increases as the temporal diversity of the channel fading available over a single packet duration decreases. For channels that exhibit a large amount of temporal diversity in the fading over a packet duration, the strong law of large numbers implies that the number of bits transmitted by the schemes of [1]–[3] for a given packet will approach its mean, which is the criterion under which the algorithms in [1]–[3] were derived. However, in most situations, such large temporal diversity will not be available, due to the delay constraints of the system. It is thus concluded that for a system with low or moderate temporal diversity of the fading over a packet length, the proposed adaptive technique does show significant gains in reducing the outage probability.

We have not shown the performance of a nonadaptive (i.e., continuous-rate) quaternary phase-shift keying (QPSK) scheme, because it requires approximately 29.2, 29.3, and 28.4 dB to achieve a packet outage probability of 0.1 for $\rho = 0.0$, $\rho = 0.8$, and $\rho = 0.99$, respectively, under the assumptions of Fig. 2. Thus, in the range displayed, nonadaptive schemes are not competitive with the adaptive algorithms, which, of course, have the significant advantage of having causal CSI available at the transmitter. However, it should be noted that the gap between the adaptive and nonadaptive schemes lessens for larger ρ and tighter delay constraints.

Finally, from Fig. 4, it is noted that the suboptimal algorithm, using a subset $(k_s = 5)$ of the original thresholds, comes very close to the performance of the optimal design scheme in terms of outage probability.

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