On the Asymptotic Capacity of MIMO Systems with Fixed Length Linear Antenna Arrays

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***Abstract*— Recently, there has been significant interest in the capacity of multiple element antenna (MEA) wireless systems. Previous authors have shown that the asymptotic capacity of a system with N transmit and N receive antennas (termed an (N, N) MEA) grows linearly with N if, for all l, the correlation of the fading for two antenna elements whose indices differ by l remains fixed as antennas are added to the array. However, in practice, the total size of the array is often fixed, and thus the correlation of the fading for two elements separated in index by some value l will change as the number of antenna elements is increased. In this paper, under the condition that the length of a linear array of antennas is fixed, the asymptotic properties of the instantaneous mutual information $I_{N,N}$ of an (N, N) MEA wireless system are derived analytically and tested for accuracy for finite N through simulations. Two different cases are considered: (1) when the fixed array size constraint is imposed at the mobile unit, and (2) when the fixed array size constraint is imposed at both the base station and the mobile unit. For the first case, simulation results indicate that the analytical approximations are very accurate for moderate values of N, especially at high signal-to-noise-ratios (SNR). For the second case, the predicted non-convergence of $I_{N,N}$ is observed in simulations, as well.**

I. Introduction

Recently, multiple element antenna (MEA) wireless systems have drawn considerable attention. Early work often assumed that for a (N_T, N_R) MEA system, with N_T transmit antennas and N_R receive antennas, the fading affecting different antenna elements was identically and independently distributed (i.i.d) as circularly symmetric complex Gaussian random variables. Under this assumption, it has been shown [1] [2] that, even if the transmitter has no knowledge of the channel fading values, the capacity divided by $\hat{N} = \min(N_T, N_R)$ approaches a non-zero constant for a fixed average transmit power, as $\hat{N} \to \infty$.

However, the aforementioned assumption of an i.i.d distribution of channel path gains can often be violated due to the insufficient spacing of antennas or the absence of a rich scattering environment around the transmitter and/or receiver. Therefore, it is of interest in many application scenarios to consider the case when the fading between different antenna pairs in a MEA wireless system is correlated. Recent work investigating the impact of correlated fading on the capacity of MEA systems can be found in [3] and [4]. In [4], a (N, N) MEA wireless system is assumed, and antennas are arranged in a regular grid, the total size of which scales upward with the number of antennas, thus preserving the relative position of adjacent antennas. Under this assumption, [4] employed random matrix theory to show that, as N approaches infinity, the instantaneous mutual information $I_{N,N}$ of such MEA systems still increases linearly, albeit with a smaller rate than in the i.i.d fading case.

In practice, while increasing the number of antennas, the entire physical size of the antenna array must often be kept fixed due to physical constraints imposed by the application. For example, on a mobile unit, the size of the antenna array cannot be assumed to grow without bound. In this work, the asymptotic characteristics of $I_{N,N}$ are investigated in a scenario where the length of a linear array is fixed and the spatial correlation function of the fading is bandlimited or satisfies certain analytic properties. Due to the constraint imposed by the fixed array site, the approaches taken in [4] cannot be employed here. Instead, the asymptotic characteristics of eigenvalues of large Hermitian matrices, as well as the statistical characteristics of eigenvalues of the large sample covariance matrices, are exploited to arrive at simple yet accurate expressions to which the asymptotic capacity will converge almost surely.

This paper is organized as follows. In Section II, the system model and channel model are presented. In Section III, asymptotic characteristics of $I_{N,N}$ are analyzed for the two cases of interest: (1) correlation exists due only to the fixed length of the array on the mobile unit, and (2) antenna arrays at both the base station and mobile unit are of fixed length. In Section IV, the same problem is reconsidered under an alternate formulation which keeps the total *received* power fixed. Simulation results are presented in Section V, and conclusions are drawn in Section VI.

II. System Model

Throughout the paper, the following notations will be used: I_N for the $N \times N$ identity matrix, $A^\dagger$ for transpose conjugate of the matrix A, A^* for conjugate of the matrix A, $\det(A)$ for determinant of the square matrix A, A' for transpose of the matrix A, and $\underline{X}$ for column vector.

As in [4], a single-user, point-to-point, narrowband wireless communication system with N transmit antennas and N

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receive antennas will be assumed through this paper. It is assumed that the transmitter has no knowledge of the channel state information (CSI) nor knowledge of the correlation statistics of the channel, but that the receiver has perfect knowledge of the CSI. Let H be the $N \times N$ channel fading matrix, whose (i,j)th entry $H_{i,j}$ is the complex path gain between transmitter j and receiver i. Then, the discrete-time equivalent system model is given by:

$$\underline{Y} = H\underline{X} + \underline{Z} \tag{1}$$

where $\underline{X}$ is an $N \times 1$ column vector whose jth component represents the signal transmitted by the jth antenna. Similarly, the received signal and received noise are represented by $N \times 1$ complex column vectors, $\underline{Y}$ and $\underline{Z}$, respectively. It is assumed that the total average power transmitted across the N transmit antennas is fixed at ρ, regardless of N. The noise vector $\underline{Z}$ is an additive white Gaussian random vector, whose entries $\{Z_i, i = 1, \ldots, N\}$ are i.i.d circularly symmetric complex Gaussian random variables with mean zero. Without loss of generality, the variance of Z_i will be normalized to one. Thus, $Z_i \sim \tilde{N}(0,1)$, where the notation $\tilde{N}(\mu, \sigma^2)$ means the variable possesses a circularly symmetric complex Gaussian distribution with mean μ and variance σ^2.

Entries of the channel fading matrix H are assumed to be circularly symmetric complex Gaussian random variables with zero mean and $E[|H_{i,j}|^2] = 1$, and thus a Rayleigh fading channel is being assumed, and the average received power at each receive antenna is ρ. In this work, H will be treated as quasi-static. Hence, as in [2], if the input vector $\underline{X}$ is a proper complex Gaussian random vector, whose covariance matrix is $E[\underline{X} \cdot \underline{X}^\dagger] = Q$, the mutual information $I_{N,N}$ of this MEA system (conditioned on H) is

$$I_{N,N} = \log_2 \det \left(I_N + H \cdot Q \cdot H^\dagger \right) \text{ bps/Hz} \tag{2}$$

Since there is no CSI nor knowledge of the correlation of entries of H available at the transmitter, a minimax robust strategy [5] is to set Q as $\frac{\rho}{N} I_N$, which implies transmitting data independently with the same average power ρ/N across each of the N antennas. Then, (2) simplifies to

$$I_{N,N} = \log_2 \det \left(I_N + \frac{\rho}{N} H \cdot H^\dagger \right) \text{ bps/Hz}. \tag{3}$$

It is assumed that the covariance matrix of the random variables $H_{i,j}$ has the following general covariance structure, as described in [4]: $E[H_{i,k} H^*_{j,l}] = \Psi^T_{k,l} \Psi^R_{i,j}$, where Ψ^T and Ψ^R are $N \times N$ covariance matrices generated by the transmit and receive antennas, respectively. Therefore, the covariance matrix of each row of H is Ψ^T, and Ψ^R is the covariance matrix of each column of H.

In [4], it was assumed that as N was increased, the relative position of adjacent antennas is fixed for some regular arrays, such as square or linear grids. Then, the empirical distribution of the eigenvalues of Ψ^T and Ψ^R are required to converge to some limiting distributions. Here, it will be assumed that both the transmit and the receive antenna are arranged as linear arrays. In contrast to [4], assume that the total length of the linear array at the receiver (mobile unit) side is fixed. The length of the linear array at the transmitter side (base station) will be assumed to be either: (1) fixed, or (2) large enough to make $\Psi^T = I_N$.

Hence, H can be factorized in the form of $H = \left(\Psi^R\right)^{\frac{1}{2}} W \left(\Psi^T\right)'^{\frac{1}{2}}$, where the entries of W are i.i.d with $\tilde{N}(0,1)$. In order to analyze the asymptotic performance of (3), as $N \to \infty$, the unitary transformation of matrices [4] yields

$$\begin{aligned} I_{N,N} &= \log_2 \det \left(I_N + \frac{\rho}{N} \Psi^R W \left(\Psi^T\right)' W^\dagger \right) \\ &\overset{\mathcal{D}}{=} \log_2 \det \left(I_N + \frac{\rho}{N} D_R W D_T W^\dagger \right) \end{aligned} \tag{4}$$

where $\mathcal{D}$ means "in distribution", and D_R and D_T are diagonal matrices, whose entries are the eigenvalues of Ψ^R and Ψ^T, respectively, in decreasing order.

III. Asymptotic Analysis of $I_{N,N}$

A. Fixed Receive Array Size with Bandlimited Power Spectral Density

In this section, it will be assumed that as $N \to \infty$, Ψ^T can be maintained as I_N. However, at the receiver side, the antennas will need to be fit into a fixed-length linear array. In such a scenario, the asymptotic characteristics of $I_{N,N}$, which is now

$$I_{N,N} = \log_2 \det \left(I_N + \frac{\rho}{N} D_R^{1/2} W W^\dagger D_R^{1/2} \right), \tag{5}$$

will be investigated.

Without loss of generality, let $\psi^R(r)$ be the normalized spatial correlation function at the receiver end for a linear array of fixed length, such that

$$\Psi^R_{i,j} = \psi^R \left(\frac{i-j}{N-1} L_R \right), \; i,j = 1, \ldots, N, \tag{6}$$

where L_R is the total length of the linear array. Therefore, Ψ^R is a non-negative definite Hermitian and Toeplitz matrix.

As noted in [11], the eigenvalues $\left\{ \lambda_k^{(R,N)} \right\}$ of the matrix Ψ^R/N will be converging to the point spectrum (i.e. eigenvalues in this case) $\left\{ \lambda_k^{(R,\infty)} \right\}$ of the non-negative definite Hermitian operator $\psi^R(x,y) = \psi^R\left[(x-y)L_R\right]$ on the Hilbert space $L_2[0,1]$, where $x, y \in [0,1]$, and can be determined by

$$\int_0^1 \psi^R \left[(x-y)L_R\right] \phi_k(y) dy = \lambda_k^{(R,\infty)} \phi_k(x), \; k = 1, \cdots, \infty, \tag{7}$$

where $\{\phi_k(x)\}$ are the eigen-functions of the operator $\psi^R(x,y)$. It has been shown in [6] that the number of nonzero values in $\left\{ \lambda_k^{(R,N)}, k = 1, \cdots N \right\}$ will be in the order of $f_1(N)$, such that $f_1(N)/N \to 0$ as N is approaching infinity.

Let $F_R(\Omega)$ be the power spectral density corresponding to $\psi^R(r)$, and assume the support of $F_R(\Omega)$ is on the interval $[-\Omega_0^R, \Omega_0^R]$, which indicates that the power spectral densities of the spatial correlation functions are assumed to be bandlimited, and is true in many applications [3]. Then, based on Toeplitz matrix theory [8], by taking the same approach as that in our work on power control [9], it can be shown that as $N \to \infty$, the eigenvalues of Ψ^R/N will be asymptotically equally distributed with the sampling points of $F_R(\Omega) \otimes F_W(\Omega)/L_R$, where $\otimes$ denotes convolution, and $F_W(\Omega)$ is the Fourier transform of the window function $W(r)$, which is 1 over $-L_R \leq r \leq L_R$, and zero otherwise.

Let $f^{\infty}(\Omega) = \frac{1}{L_R}\int_{-L_R}^{L_R} \psi_R(r) e^{-j\Omega r} dr$. Since $f^{(\infty)}(\Omega)$ is the Fourier transform of the spatial correlation function *truncated* to $[-L_R, L_R]$, the strictly bandlimited nature of $F_R(\Omega)$ implies that $f^{(\infty)}(\Omega)$ is not bandlimited. However, as will be shown below, the numerical results suggest that a bandlimited approximation to such is quite useful for numbers of antenna elements of interest. In particular, the power spectral density decays rapidly outside of a transition region. Thus, while the $\left\{\lambda_k^{(R,N)}\right\}$ are converging point-wisely to the point spectrum $\left\{\lambda_k^{(R,\infty)}\right\}$ of the Hermitian and Toeplitz operator specified in (7), the number of nonzero eigenvalues $f_1(N)$ can be approximated as a finite number N_R. In other words, a finite number N_R of nonzero bounded eigenvalues $\{\lambda_k^{(R,N)}, k = 0, \ldots, N_R - 1\}$ will dominate, and all other $N - N_R$ eigenvalues can be approximated as zero, where N_R is fixed and $\lambda_k^{(R,N)} \to \lambda_k^{(R,\infty)}$ when N is big. It should be noted that *any* non-zero eigenvalue will eventually (N large enough) have a significant absolute impact on the capacity, but this does not happen until N approaches the inverse of that eigenvalue, and thus the threshold below which a sample of the power spectral density is ignored can be chosen small enough to place those N beyond the values of interest. Since the applicability of this entire line of research is predicated upon the point-wise convergence of $\left\{\lambda_k^{(R,N)}\right\}$ to the point spectrum $\left\{\lambda_k^{(R,\infty)}\right\}$ of the Hermitian and Toeplitz operator at reasonable N, this causes no conceptual problem, and the numerical results will firmly support this approach. In [6], it has been shown as well that even if $\psi^R(r)$ is not bandlimited, as long as $\psi^R(r)$ satisfies certain analytical conditions such that $\lambda_k^{(R,\infty)}$ decreases fast enough [10], $f_1(N)$ can still be approximated as N_R.

Therefore, with $f_1(N) \approx N_R < \infty$, as $N \to \infty$, the diagonal matrix D_R/N in (5) can be approximated as a diagonal matrix whose N_R upper left diagonal entries are positive, and all other entries will be vanishing. Hence, (5) becomes

$$I_{N,N} \approx \hat{I}_{N_R,N} = \log_2 \det\left(I_{N_R} + \rho \hat{D}_R^{1/2} \hat{W} \hat{W}^{\dagger} \hat{D}_R^{1/2}\right) \quad (8)$$

where $\hat{D}_R$ is a $N_R \times N_R$ diagonal matrix with diagonal entries as $\{\lambda_k^{(R,N)}\}$, and $\hat{W}$ is a $N_R \times N$ random matrix, whose entries are circularly symmetric complex Gaussian random variables: $\sim$ i.i.d $\tilde{N}(0,1)$.

$$\lim_{N\to\infty}\left(\hat{I}_{N_R,N} - \sum_{i=0}^{N_R-1} \log_2\left(1 + N\rho\lambda_i^{(R,\infty)}\right)\right) = 0, \text{ a.s.} \quad (9)$$

where a.s. means almost sure convergence. See [6] and [7] for the proof of (9).

Convergence in (9) implies that for a (N,N) MEA system with spatial correlation only at the mobile unit side with bandlimited power spectral density, the instantaneous mutual information of the MEA system $I_{N,N}$ can be approximated as $\hat{I}_{N_R,N}$, whose difference with the sum term in (9) diminishes almost surely. Note that, as N increases, $I_{N,N}$ is increasing at the speed of $\log_2(N)$ in the high SNR case, which is in contrast to the case without the length constraint at the receiver end, where it was shown that $I_{N,N}$ will be increasing at the speed of N (i.e. linearly) as proved in [4]. Due to the fast decreasing rate to zero of the eigenvalues of the spatial correlation function, the (N,N) MEA system is roughly equivalent to a (N,N_R) MEA system, with transmit power allocated the same as ρ at each transmit antenna, where N_R is a fixed number, as N is sufficiently large. Furthermore, this (N,N_R) MEA system can be decomposed into $N_R \times N_R$ parallel independent channels with receive SNR $N\rho\lambda_k^{(R,\infty)}, k = 0, \ldots, N_R - 1$. Therefore, N_R can roughly be considered the *degrees of freedom* as defined in [3].

B. Bandlimited Spatial Correlations at Transmitter and Receiver

In this section, the case with fixed length linear arrays located at both the base station and mobile unit will be investigated. In this case, let $\psi^R(r)$ and $\psi^T(r)$ be the spatial correlation functions, and let L_R and L_T be the length of the linear arrays, respectively. By following the same approach taken above, it can be concluded that there are N_R and N_T dominant eigenvalues of matrices Ψ^R/N and Ψ^T/N, respectively, which are denoted as $\hat{D}_R = \text{diag}\left\{\lambda_i^{(R,N)}, i = 0, \ldots, N_R - 1\right\}$ and $\hat{D}_T = \text{diag}\left\{\lambda_i^{(T,N)}, i = 0, \ldots, N_T - 1\right\}$, respectively. Therefore, for N sufficiently large, (4) becomes:

$$\begin{aligned} I_{N,N} &\overset{\mathcal{D}}{=} \log_2 \det\left(I_N + \frac{\rho}{N} D_R W D_T W^{\dagger}\right) \\ &\approx \log_2 \det\left(I_{N_R} + N\rho \hat{D}_R \tilde{W} \hat{D}_T \tilde{W}^{\dagger}\right) \quad (10) \end{aligned}$$

where $\tilde{W}$ is a $N_R \times N_T$ random matrix, whose entries are i.i.d and distributed as $\tilde{N}(0,1)$, and N_R and N_T are fixed numbers. From (10), it can be seen that such a (N,N) MEA system is equivalent to a (N_R, N_T) MEA system in terms of the statistical behavior of $I_{N,N}$ when N is sufficiently large, with spatial correlations determined by $\hat{D}_R$ and $\hat{D}_T$, and with transmit power at each transmit antenna of $N\rho$. Since the dimension of $\tilde{W}$ is finite, $I_{N,N}$ is a random variable regardless of the size of N. The statistical characteristics of $I_{N,N}$ will be subject to the probability distribution function of the eigenvalues of matrix $\hat{D}_R \tilde{W} \hat{D}_T \tilde{W}^\dagger$. As one might expect, it can be shown [6] that the expected value of $I_{N,N}$ in (10) can be upper bounded by $I_{N,N}$ with the fixed length linear array at the receiver end only (i.e. the sum term in (9)).

IV. Case with Fixed Total Received Signal Power

In previous sections, it is assumed the total transmitted power is ρ after normalizing the variance of the additive noise. In that case, the total signal power received by the receiving antennas scales with the number of receive antennas N. However, in recent independent work [11], which studied ergodic (or mean) capacity (rather than the characterization of each sample path) under a similar framework, it is argued that the total received signal power should be kept constant if multiple antennas occupy a given physical volume. This is accomplished mathematically by scaling the transmitted power by $1/N$, thus making the correlation matrix Q in (2) equal to $\frac{\rho}{N^2} I_N$. Based on this assumption, in [11], they argued that if there exists only one-sided correlation caused by the receive antennas, $E\left[I_{N,N}\right]$ (i.e. average of $I_{N,N}$ in (2)) is converging to a constant.

In this section, by employing the same approach as that in Section III, a stronger result than that in [11] will be shown. In particular, with a fixed length linear array on the receiver side, as well as the total received signal power fixed, the instantaneous mutual information $I_{N,N}$ will be shown to converge almost surely to a deterministic constant as $N \to \infty$.

Theorem 1:

$$\begin{aligned} I_{N,N} &= \log_2 \det \left(I_N + \frac{\rho}{N^2} D_R^{1/2} W W^\dagger D_R^{1/2} \right) \\ &\overset{\text{a.s.}}{\longrightarrow} \sum_{k=0}^{f_1(N)-1} \log_2 \left(1 + \rho \lambda_k^{(R,\infty)} \right) \le \frac{\rho}{\ln 2}. \end{aligned} \quad (11)$$

Proof: See [6]. The proof of Theorem 1 does not require the condition that $\psi^R(r)$ is bandlimited or has nice analytical properties [6]. We note that a similar result to Theorem 1 has recently been stated in independent work [12], without explicitly stating the almost sure convergence, though.

If the transmitting antennas are also spatially dense, (e.g. with fixed length linear arrays located at both the base station and mobile), and $Q = \frac{\rho}{N^2} I_N$, it can be shown as well that [6],

$$\begin{aligned} E\left[I_{N,N}\right] &\le \sum_{k=0}^{N-1} \log_2 \left(1 + \rho \lambda_k^{(R,N)} \right), \text{ for all } N \\ &\longrightarrow \sum_{k=0}^{f_1(N)-1} \log_2 \left(1 + \rho \lambda_k^{(R,\infty)} \right), \text{ as } N \to \infty \end{aligned} \quad (12)$$

which indicates that the expected value of $I_{N,N}$ in this scenario is upper bounded by the capacity of the MEA system with the fixed length linear array located at receiver side only, asymptotically. As more antennas are put in this MEA system, the ergodic capacity will be non-decreasing under the conditions assumed throughout this work; since $E\left[I_{N,N}\right]$ has a finite upper bound, it can be concluded that

$$E\left[I_{N,N}\right] \to C, \quad (13)$$

which agrees with what is claimed in [11] for the mean capacity, and C is a finite constant that will depend on the spectrum of the Hermitian operators, $\psi^R(x,y)$ and $\psi^T(x,y)$, respectively.

V. Simulation Results

To determine how well the analytical results of Section III hold for finite N and choice of N_R (which corresponds to a selection of the "dominant" eigenvalues), as well as Theorem 1 in Section IV, a number of simulation results are presented here. The parameters employed are as follows: linear array at the mobile unit of length $L_R = 5\lambda$, where λ is the carrier wavelength, and signal-to-noise ratios (SNR) of $\rho = 22$ dB. Let the correlation function be given by $\psi_1^R(r) = e^{-1/2(2\pi r \sigma_\theta/\lambda)^2}$, where $\sigma_\theta = 0.25$, $\psi_2^R(r) = \text{sinc}\,(2r/\lambda)$ [3] (i.e. bandlimited), and $\psi_3^R(r) = e^{-|r|/\lambda}$. For the case with a fixed length array at both transmitter and receiver, let $\psi^R(r) = \psi^T(r)$.

If the total average transmit power is fixed, Figure 1 and Figure 2 show the characteristics of $I_{N,N}$ versus N, with spatial correlation function $\psi_1^R(r)$ and $\psi_2^R(r)$, respectively. The number of dominant eigenvalues N_R in both cases are 15, which has been determined through numerical computations. In those figures, analytical results are obtained with the sum term in (9); simulation results are obtained through generating one realization of the random variable $I_{N,N}$ for each N in terms of the first equation in (4). From the simulation results, it can be observed that the sum term in (9) is a very accurate approximation for $I_{N,N}$ if there exists bandlimited spatial correlation at one side or the spatial correlation function is smooth enough. If fixed length arrarys are located at both sides, $I_{N,N}$ does not show convergence as expected.

If the total average received power is fixed, which can be realized mathematically by setting $Q = \frac{\rho}{N^2} I_N$ as argued in Section IV, Figure 3 shows the results using the spatial correlation function $\psi_3^R(r)$, which does not have a derivative at $r = 0$. It can be observed that (11) agrees well with the simulation result. However, it has been shown in [6] that this convergence is much slower than those of $\psi_1^R(r)$ and $\psi_2^R(r)$, which have nicer analytical properties than that of $\psi_3^R(r)$.

VI. Conclusion

In this paper, the convergence of the instantaneous mutual information $I_{N,N}$ of a (N,N) MEA system is presented analytically and tested through simulations, for the case when spatial correlations are caused by the restriction that the elements of the array must occupy a fixed length at either the mobile unit or at both sides. It has been shown that the analytical result (9) is accurate to approximate $I_{N,N}$ at moderate values of N, especially in the case of high signal-to-noise ratios.

If the total received signal power is fixed by normalizing the transmitted power by $1/N$, simulation results presented agree well with the analytical ones in Section IV.

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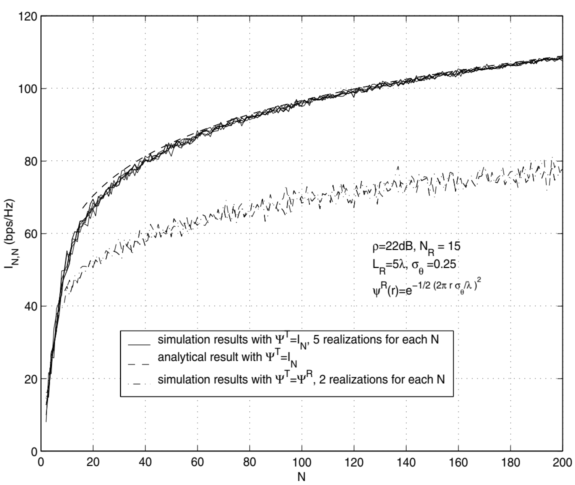

Fig. 1. Instantaneous mutual information $I_{N,N}$ of a (N, N) MEA system versus N, with the fixed total average transmit power, $\psi^R(r) = e^{-1/2(2\pi r\sigma_\theta/\lambda)^2}$, where $\sigma_\theta = 0.25$, $\rho = 22dB$, $L_R = 5\lambda$, and $f_1(N) \approx N_R = 15$.

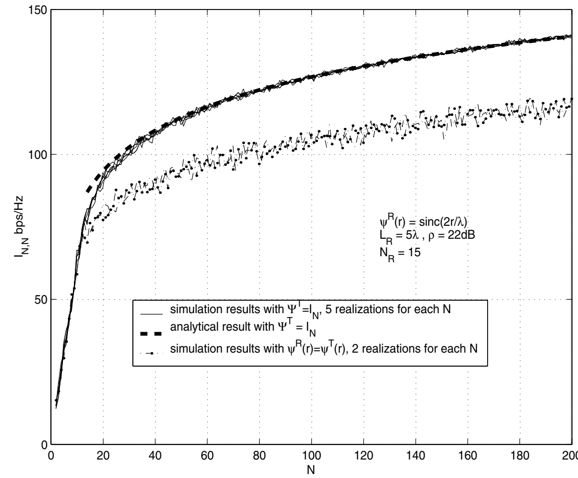

Fig. 2. Instantaneous mutual information $I_{N,N}$ of a (N, N) MEA system versus N, with the fixed total average transmit power. Simulation results are obtained through generating one realization of random variable $I_{N,N}$ for each N in terms of the first equation in (4), where $L_R = 5\lambda$, $\rho = 22$dB. For the case when the fixed length linear array is put at the receiver side only, 5 realizations are generated for each N, and the Toeplitz matrix Ψ^R is determined by $\psi^R(r) = \text{sinc}\,(2r/\lambda)$, $\Psi^T = I_N$. For the case when the fixed length linear arrays are put at both the transmitter and receiver side, 2 realizations are generated for each N, and $\Psi^R = \Psi^T$ are determined by $\psi^T(r) = \psi^R(r) = \text{sinc}\,(2r/\lambda)$. Analytical results are obtained by the sum term in (9), where $N_R = 15$ and eigenvalues $\left\{\lambda_k^{(R,N)}\right\}$ are obtained from numerical calculations. It can be observed that for each realization in the case with a fixed length array at the receiver side only, $I_{N,N}$ is converging rapidly for large N. The case of a fixed length array located at both sides is indicated by the dash-dot lines, more randomness is observed as expected.

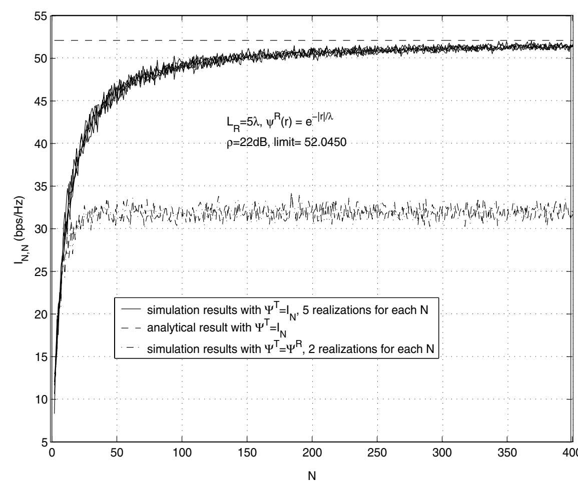

Fig. 3. Instantaneous mutual information $I_{N,N}$ of a (N, N) MEA system versus N, with fixed total average received power, such that $Q = \frac{\rho}{N^2} I_N$ in (2), and $\psi^R(r) = e^{-|r|/\lambda}$. Analytical results are obtained by the sum term in (11), where the eigenvalues $\left\{\lambda_k^{(R,N)}\right\}$ are obtained through numerical computations and $\sum_{k=0}^{f_1(400)-1} \log_2\left(1 + \rho\lambda_k^{(R,400)}\right) = 52.0460$. It can be observed that for each realization in the case with a fixed length array at the receiver side only, $I_{N,N}$ is converging to the analytical result as N grows large. The case of a fixed length array located at both sides is indicated by the dash-dot lines, more randomness is observed as expected, and the expected value demonstrates the convergence as well.