**ECE 563 - Intro to Comm. and Sig. Proc., Fall 2006**

**Midterm Exam #2**

**Wednesday, November 15th, 6:30-8:30pm, ELAB 303**

## Overview

- The exam consists of four problems for 100 points. The points for each part of each problem are given in brackets **-** you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **two page-sides** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

## Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write “this must be wrong because . . . ” so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** **-** the *minimum penalty* will be an “F” for the course.

# Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

| **Time Function** | **Fourier Transform** |
|---|---|
| $x(at+b)$ | $\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$ |
| $p(t) = \begin{cases} 1 & \lvert t\rvert \le 1/2 \\ 0 & \lvert t\rvert > 1/2 \end{cases}$ | $P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$ |
| $\cos(2\pi f_0 t)$ | $\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$ |
| $\sin(2\pi f_0 t)$ | $\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$ |
| $x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \le 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$ | $X(f) = \text{sinc}^2(f)$ |

<u>Parseval's Relation:</u> If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. A message $m(t)$ that you know is a pure sinusoid (i.e. $m(t) = A_m \cos(2\pi f_m t + \theta)$) is input to an analog communication transmitter. The output $x(t)$ of the transmitter is shown in Figure 1 below.

   [5] (a) Is this an amplitude modulation (AM) transmitter or a frequency modulation (FM) transmitter? (Be sure to justify your answer.)

   [7] (b) Give a possible message $m(t)$ (i.e. Find possible $A_m$, $f_m$, and $\theta$ for the sinusoidal message). *Note: You will have a hard time finding one of the three parameters exactly - just estimate it as best you can from the plot.*

   [8] (c) Find the modulation index $a$ (if it is AM) or $\beta$ (if it is FM).

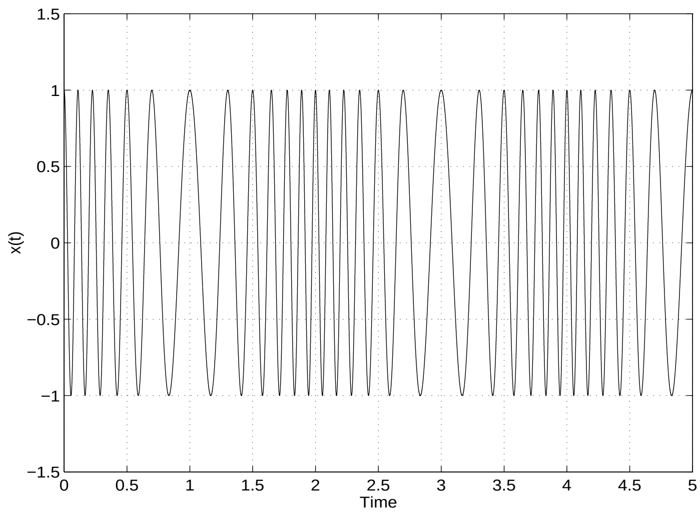
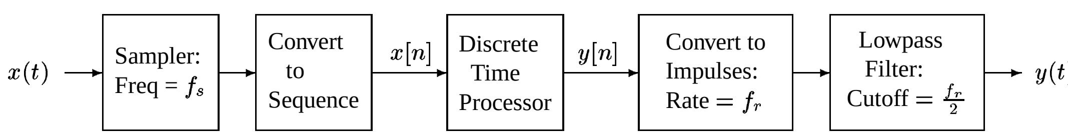



Figure 1: Signal $x(t)$ for Problem 1

2. Consider the following system to process continuous-time signals with discrete-time processing.

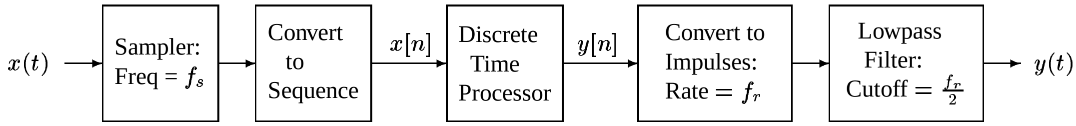



   [8] (a) Suppose that $x(t) = \cos(2\pi 3000t)$ is sampled at $f_s = 12000$ Hz to yield $x[n]$. The signal is interpolated by a factor of 2 to yield $y[n]$, which is converted back to $y(t)$ using $f_r = 24000$ Hz (the most natural recovery frequency since we have increased the sampling rate). Find $y(t)$.

   [7] (b) Suppose that $x(t) = \cos(2\pi 3000t)$ is sampled at $f_s = 12000$ Hz to yield $x[n]$. The signal is interpolated by a factor of 2 to yield $y[n]$, which is converted back to $y(t)$ using $f_r = 12000$ Hz. Find $y(t)$.

   [5] (c) Let everything between $x(t)$ and $y(t)$ in the block diagram above be termed "the system". If the only discrete-time processor operations allowed are interpolation, decimation, and linear operations $h[n]$, can we say that "the system" must be linear?

3. Consider the following system to process continuous-time signals with discrete-time processing.




Your discrete-time processing toolkit on your computer has the following five blocks for possible use (when a frequency response is given, it is valid for $\omega \in [-\pi, \pi]$, and it repeats outside of that range, of course):

**Block 1:** $H_1(e^{j\omega}) = e^{-j\omega n_0}$, where you can choose any $n_0$ that you wish

**Block 2:** $H_2(e^{j\omega}) = j\omega$

**Block 3:** $H_3(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{2} \\ 0, & \text{else} \end{cases}$

**Block 4**: The fourth block provides output $y[n] = (-1)^n x[n]$ for input $x[n]$.
**Block 5**: The fourth block provides output $y[n] = \sum_{k=0}^{n} x[k]$ for input $x[n]$.

Using any combinations of these blocks (including multiple versions of any, if needed), scalar multiplies (if needed), and specifying the lowest possible sampling frequencies $f_s$ and $f_r$ (which can be different), provide a separate system to implement each of the following desired continuous-time operations with the block diagram above.

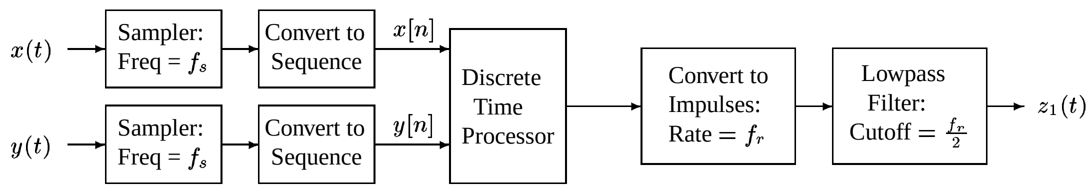
[10] (a) The lowpass filter $H(f) = p(f/30000)$, to be applied on input signals $x(t)$ of bandwidth up to 25 KHz to yield the signal $y(t)$.

[10] (b) A system that delays the input $x(t)$, of bandwidth less than 100 KHz, by $3 \times 10^{-4}$ seconds to yield the signal $y(t)$.

[10] (c) A system that yields $y(t) \approx \int_0^t x(s)ds$ for inputs $x(t)$ of bandwidth less than 100 KHz. How could you improve your approximation?

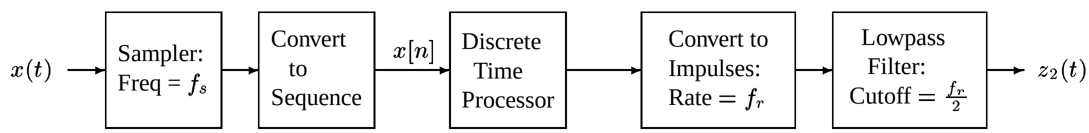
4. For each of the designs below, suppose that the power required to perform continuous-to-discrete (C/D) conversion and discrete-to-continuous (D/C) conversion dominates my power budget, and thus I want to use the smallest $f_s$ and $f_r$ possible. Find $f_s$, $f_r$, and the appropriate discrete-time processing for each of the examples below.

[10] (a) Consider a system that has two continuous-to-discrete converters running at a frequency $f_s$, a discrete-time processor, and a discrete-to-continuous converter running at frequency $f_r$:



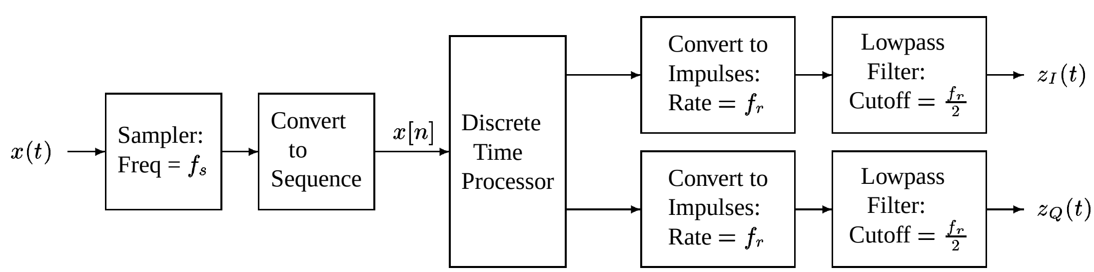

As input, I have two signals $x(t)$ and $y(t)$, each of bandwidth 10 KHz. Find $f_s$, $f_r$, and the discrete-time processing such that $z_1(t) = x(t) + y(t)$. *Remember to keep each of your sampling frequencies as low as possible.*

[10] (b) Consider a system that has a continuous-to-discrete converter running at a frequency $f_s$, a discrete-time processor, and a discrete-to-continuous converter running at frequency $f_r$:




As input, I have a signal $x(t)$ with bandwidth 100 KHz. Find $f_s$, $f_r$, and the discrete-time processing such that $z_2(t) = x^2(t)$. *Remember to keep each of your sampling frequencies as low as possible.*

[10] (c) Consider a system that has a continuous-to-discrete converter running at a frequency $f_s$, a discrete-time processor, and two discrete-to-continuous converters running at frequency $f_r$:




As input, I have a signal $x(t)$ with bandwidth 10 KHz and $\max_t |x(t)| = 4$. Find $f_s$, $f_r$ and the discrete-time processing such that:

$$
\begin{aligned}
z_I(t) &\approx A_c \cos(2\pi 5000 \int_0^t x(\tau) d\tau), \\
z_Q(t) &\approx A_c \sin(2\pi 5000 \int_0^t x(\tau) d\tau)
\end{aligned}
$$

Note that $z_I(t)$ and $z_Q(t)$ are the in-phase and quadrature signals, respectively, for an FM waveform. *Remember to keep each of your sampling frequencies as low as possible.*