ECE 563 - Fall 2003
Exam 2 (2 hours, 15 minutes; Closed Book - Two Sheets of Notes Allowed)

1. *(12%)* In a phase-locked loop (PLL) used as an FM demodulator, the PLL output signal $v(t)$ and the message signal $m(t)$ are (approximately) related by the frequency-domain equation

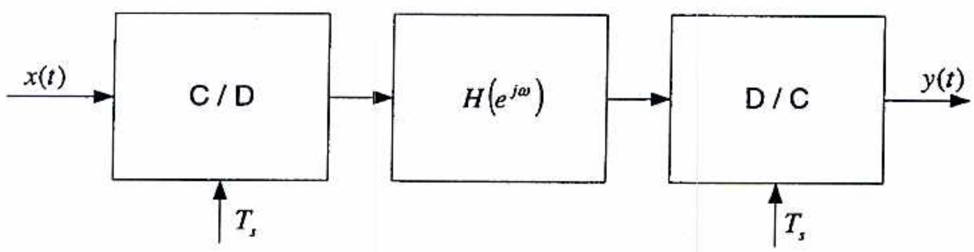
$$V(f) = \left\{ \frac{A(jf)^{-1}G(f)}{1 + B(jf)^{-1}G(f)} \right\} M(f),\ |f| \le W$$

where $G(f)$ is the loop filter frequency response, A and B are constants, and W is the message signal bandwidth.

   (a) *(6%)* Find $G(f)$, $|f| \le W$, that gives $v(t) = m(t)$.

   (b) *(6%)* In a frequency discriminator FM demodulator, there is a filter that acts like a differentiator over the transmitted signal bandwidth. Based on your result for part (a), what comparable statement can you make about the loop filter in a PLL FM demodulator?

2. *(16%)* Suppose that we want to implement an analog filter having frequency response $H(f)$ proportional to jf for $|f| \le W$. We want to do this using the system shown below:



where the discrete-time filter has frequency response $H(e^{j\omega}) = j\sin(\omega)$, $|\omega| \le \pi$.

   (a) *(8%)* Assuming that $\sin(\omega) \approx \omega$ for $|\omega| \le 0.1\pi$ and that $W = 15$ kHz, what range of values can we use for T_s?

   (b) *(8%)* Recalling that $j\sin(\omega) = \frac{1}{2}\{e^{j\omega} - e^{-j\omega}\}$, find and sketch the impulse response of the discrete-time filter of part (a).

3. *(22%)* Consider a discrete-time filter having frequency response:

$$H(e^{j\omega}) = \frac{1 - 2e^{-j\omega}}{1 + 0.5e^{-j\omega}}$$

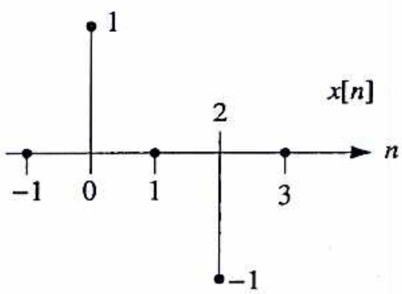
   (a) *(5%)* Find a difference equation relating the filter input $x[n]$ and output $y[n]$.

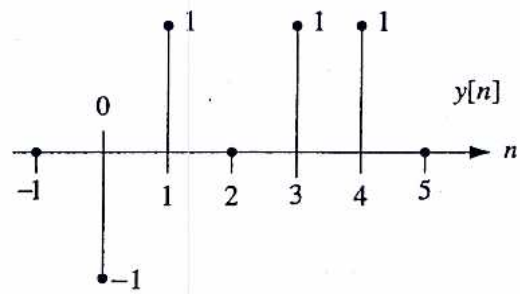
   (b) *(5%)* Draw the block diagram of a tapped delay line filter implementation.

(c) *(7%)* Find the filter's impulse response. (*Hint:* Use the Fourier Transform tables attached to this exam questionnaire.)

(d) *(5%)* Is the filter stable? (Justify your answer.)

4. *(16%)* For the discrete-time signals $x[n]$ and $y[n]$ shown below:





(a) *(8%)* Find and sketch $z_1[n]$ = the 5-point circular convolution of $x[n]$ and $y[n]$ for $0 \le n \le 7$.

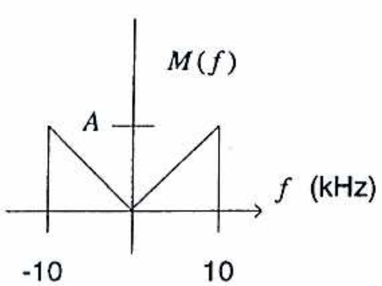
(b) *(8%)* Find and sketch $z_2[n] = x[n] * y[n]$ (the regular convolution of $x[n]$ and $y[n]$) for $0 \le n \le 7$.

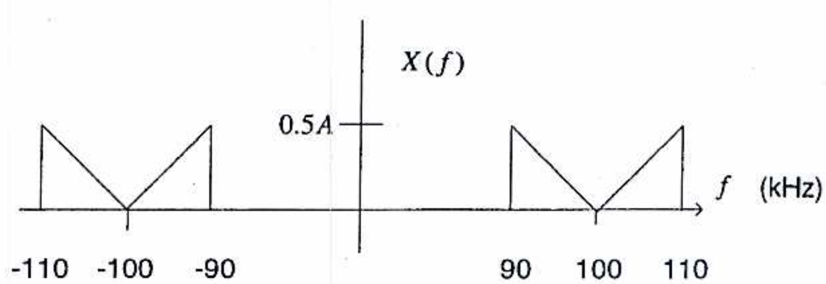
5. *(12%)* Suppose we have a signal that is bandlimited to 5 kHz. We sample the signal at a rate of 12000 samples/sec. We want to examine the signal's frequency components using a DFT.

(a) *(6%)* If the DFT length must be a power of 2 and the analog frequency spacing between adjacent DFT samples can be no more than 5 Hz, what is the minimum DFT length N that can be used?

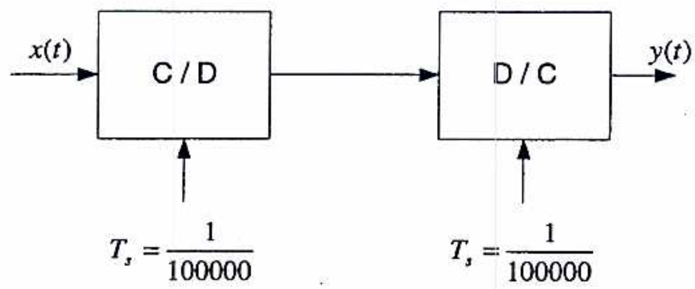
(b) *(6%)* For the N that you found in part (a): what DFT sample number k is closest to representing an analog frequency of 4600 Hz?

6. *(22%)* Suppose we have a message signal $m(t)$ that is bandlimited to 10 kHz. Its Fourier Transform $M(f)$ is shown below. This signal is used to form the DSBSC signal $x(t)$ having a carrier frequency of 100 kHz. The Fourier Transform $X(f)$ of the DSBSC signal is also shown below.



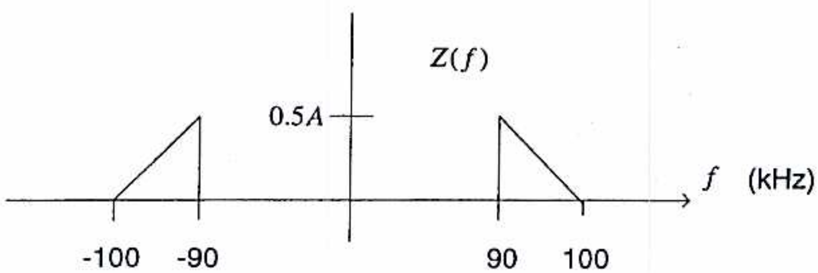


(a) *(12%)* Consider the system:

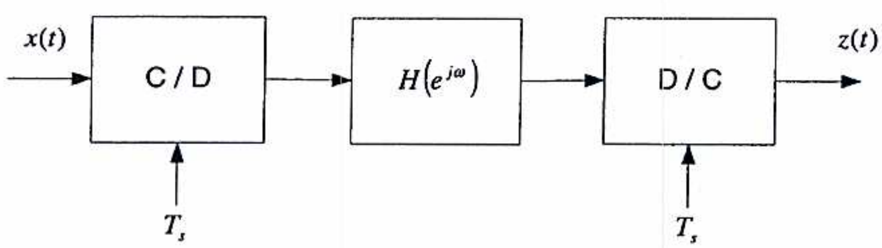


where $x(t)$ is the DSBSC signal described above. Find the system output $y(t)$ in terms of $m(t)$. What analog system does this implement?

(b) *(10%)* Now suppose that we want to process the DSBSC signal $x(t)$ to form the LSSB signal $z(t)$ whose Fourier Transform $Z(f)$ is shown below:



We want to implement the processing with the system shown below:



Find a specific T_s and $H(e^{j\omega})$ that implement the desired processing.

TABLE 2.2 FOURIER TRANSFORM THEOREMS

Sequence	Fourier Transform
$x[n]$ $y[n]$	$X(e^{j\omega})$ $Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

Parseval's theorem:

8. $$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

9. $$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$$

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n - n_0]$	$e^{-j\omega n_0}$
3. 1 $(-\infty < n < \infty)$	$\displaystyle\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n]$ $(\|a\| < 1)$	$\dfrac{1}{1 - ae^{-j\omega}}$
5. $u[n]$	$\dfrac{1}{1 - e^{-j\omega}} + \displaystyle\sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n+1)a^n u[n]$ $(\|a\| < 1)$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$
7. $\dfrac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n]$ $(\|r\| < 1)$	$\dfrac{1}{1 - 2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\dfrac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \|\omega\| < \omega_c, \\ 0, & \omega_c < \|\omega\| \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\displaystyle\sum_{k=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\displaystyle\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$