ECE 563 - Intro to Comm. and Sig. Proc., Fall 2001

Midterm Exam #2

Wednesday, November 14th, 6:00-8:00, GOES 20

Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **two page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	Fourier Transform
$x(at+b)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right) e^{j2\pi \frac{b}{a} f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
$x(t) = \begin{cases} 1 - \lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. **Note: This problem is not as hard as you might think at first. Even if you are not able to get the whole sequence, get as much as you can for partial credit!**

   Let $x[n]$ be a purely real sequence. You work in the laboratory to obtain the following information about $x[n]$:

   (i) $x[n]$ is a causal sequence.

   (ii) If $v[n] = x[n+2]$, the Fourier transform $V(e^{j\omega})$ of the discrete-time sequence $v[n]$ is purely real.

   (iii)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 16$$

   (iv)

$$x[0] = 2$$

   (v)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega} d\omega = 1.$$

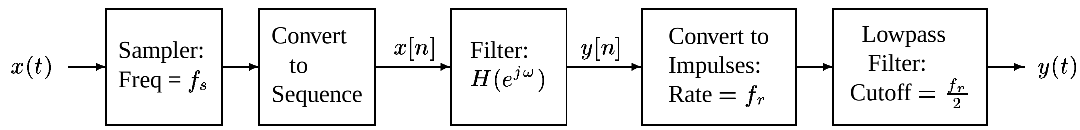
   (vi) $x[2] > 0.$

   [15] Find the discrete-time sequence $x[n]$.

2. Many amplitude modulation (AM) schemes can be implemented through sampling and filtering. You are given a message signal $m(t)$ that is low-pass with bandwidth 20 KHz and desire to form the DSB-SC signal $x(t) = A_c m(t)\cos(2\pi 10^6 t)$. Suppose that you have an ideal sampler laying around the lab; that is, you have a sampler that gives the output $y(t) = x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t-kT)$ in response to the input $x(t)$, where T is a user-programmable sampling period.

   [15] Give the block diagram for a system that takes $m(t)$ as input and yields $x(t)$ as output. You can employ your ideal sampler (be sure to specify T) and any linear time-invariant filters you wish.

3. Consider the following system to process continuous-time signals with discrete-time processing (Note: This is the same system we have been using in class, with sampling rate f_s for the C/D and impulse rate f_r for the D/C).



The discrete-time filter frequency response, which you recall is periodic with period 2π, is given in $[-\pi, \pi]$ by:

$$H(e^{j\omega}) = |\omega|, \qquad |\omega| \leq \pi$$

[10] (a) Suppose that the signal $x(t) = 5\cos(2\pi 5000t)$ is input to the system. Find the system output $y(t)$ if $f_s = 15$ kHz and $f_r = 15$ kHz.

[10] (b) Suppose that the signal $x(t) = 5\cos(2\pi 5000t)$ is input to the system. Find the system output $y(t)$ if $f_s = 7.5$ kHz and $f_r = 7.5$ kHz.

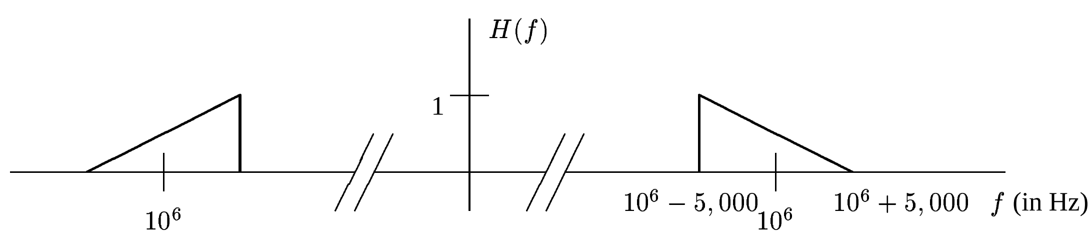
[5] (c) Suppose that the signal $x(t) = 5\cos(2\pi 5000t)$ is input to the system. Find a sampling frequency f_s and C/D impulse rate f_r such that the output consists of only a tone at 1 KHz.

4. In your first job after graduation from UMass, you are assigned to design a transmitter that outputs $y(t) = h(t) * x(t)$, where

$$x(t) = x_I(t)\cos(2\pi 10^6 t) - x_Q(t)\sin(2\pi 10^6 t)$$

is a bandpass signal with lowpass signals $x_I(t)$ and $x_Q(t)$, (each of bandwidth 5 KHz) as its in-phase and quadrature components, respectively, and $h(t)$ is a bandpass filter. The inputs to your transmitter are $x_I(t)$ and $x_Q(t)$, and the output is $y(t)$.

[10] (a) Suppose that the (real) bandpass filter response is as given below, where $H(f)$ is the Fourier transform of $h(t)$:



Sketch the Fourier transforms $H_I(f)$ and $H_Q(f)$ of the in-phase part $h_I(t)$ and quadrature part $h_Q(t)$, respectively, of the filter $h(t)$. *Hint:* Recall from class that

$$H_I(f) = \frac{H_Z(f) + H_Z^*(-f)}{2}$$
$$H_Q(f) = \frac{H_Z(f) - H_Z^*(-f)}{2j},$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

[5] (b) Draw a circuit that takes as input $x_I(t)$ and $x_Q(t)$ and outputs $x(t)$, while employing only summers, multipliers, oscillators, and analog *lowpass* filters.(*Note:* A "lowpass filter" is defined for this problem as one whose frequency response is non-zero only for $|f| \leq 5$ KHz).

[10] (c) Now suppose that we want to filter the signals $x_I(t)$ and $x_Q(t)$ with *digital* lowpass filters rather than analog lowpass filters. Specify the circuitry to replace your analog lowpass filters in part (b) with C/D converters, digital filters, and D/C converters, being sure to specify all sampling rates and digital filter responses (either in the time or frequency domain is okay). **Note: If you cannot get part (a), just make up some reasonable 5 KHz analog filter shapes and do part (c) anyway!**