

ECE 563 - Intro to Comm. and Sig. Proc., Fall 2006

Midterm Exam #1

Tuesday, October 17th, 6:30-8:30pm, Marston 132

Overview

- The exam consists of four problems for 100 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write “this must be wrong because ...” so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an “F” for the course.

Some potentially useful information

$$\begin{aligned}\cos(\theta) &= \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \\ \sin(\theta) &= \frac{1}{2j} (e^{j\theta} - e^{-j\theta})\end{aligned}$$

$$\begin{aligned}\sin(a \pm b) &= \sin(a) \cos(b) \pm \cos(a) \sin(b) \\ \cos(a \pm b) &= \cos(a) \cos(b) \mp \sin(a) \sin(b)\end{aligned}$$

$$\begin{aligned}\cos(a) \cos(b) &= \frac{1}{2} [\cos(a - b) + \cos(a + b)] \\ \sin(a) \sin(b) &= \frac{1}{2} [\cos(a - b) - \cos(a + b)] \\ \sin(a) \cos(b) &= \frac{1}{2} [\sin(a - b) + \sin(a + b)]\end{aligned}$$

Time Function	Fourier Transform
$x(at + b)$	$\frac{1}{ a } X\left(\frac{f}{a}\right) e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & t \leq 1/2 \\ 0 & t > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j} \delta(f - f_0) - \frac{1}{2j} \delta(f + f_0)$
$x(t) = \begin{cases} 1 - t & t \leq 1 \\ 0 & t > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

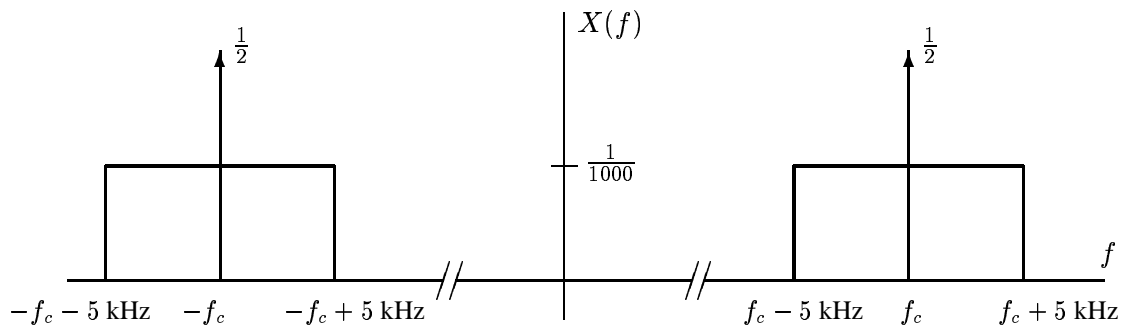
$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. *Basic Frequency Domain:*

[10] (a) Consider the DSB-SC signal $x(t) = A_c m(t) \cos(2\pi f_c t + \theta)$, where $m(t)$ is a real lowpass message signal with Fourier transform $M(f)$ and bandwidth $W \ll f_c$. Use the properties of the Fourier transform to derive $X(f)$, the Fourier transform of $x(t)$, in terms of $M(f)$. *Note: The key here is the derivation, so providing the answer without justification will earn very little credit.*

[10] (b) Consider the convolution of a signal $a(t)$ with itself to yield $b(t) = a(t) * a(t)$. Does there exist a signal $a(t)$ such that $a(t) = b(t) = a(t) * a(t)$? *Note: Be sure to justify your answer.*

2. An amplitude modulation (AM) transmitter outputs a signal $x(t)$ with Fourier transform $X(f)$ given by:



The carrier frequency is $f_c = 1$ MHz. You also know that the message spectrum $M(f)$ contains no $\delta(\cdot)$ functions.

[10] (a) Find $x(t)$ and then *roughly* sketch it. [In your drawing, focus on conveying the key points rather than artistic excellence.]

[5] (b) Find the bandwidth of the signal $x(t)$.

[7] (c) Provide the simplest possible receiver that can be used to recover $m(t)$. Be sure to provide all necessary parameters (e.g. filter bandwidths, oscillator frequencies, critical component values, etc.).

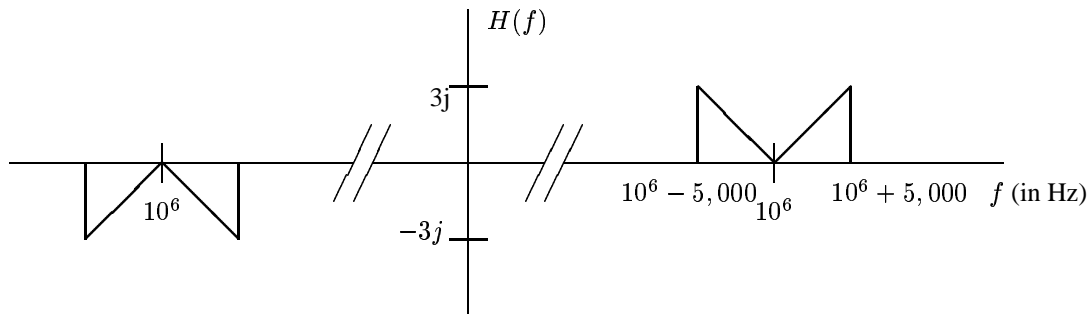
[8] (d) Suppose I remove the unmodulated carrier component from $x(t)$, leaving just a term of the form $A_2 m(t) \cos(2\pi f_c t)$. Find the energy in the resulting signal.

3. In your first job after graduation from UMass, you are assigned to design a transmitter that outputs $y(t) = h(t) * x(t)$, where

$$x(t) = x_I(t) \cos(2\pi 10^6 t) - x_Q(t) \sin(2\pi 10^6 t)$$

is a bandpass signal with lowpass signals $x_I(t)$ and $x_Q(t)$ (each of bandwidth 5 KHz) as its in-phase and quadrature components, respectively, and $h(t)$ is a real bandpass filter. The inputs to your transmitter are $x_I(t)$ and $x_Q(t)$, and the output is $y(t)$.

- [10] (a) Suppose that the bandpass filter response is specified by $H(f)$, the Fourier transform of $h(t)$:



Sketch the Fourier transforms $H_I(f)$ and $H_Q(f)$ of the in-phase part $h_I(t)$ and quadrature part $h_Q(t)$, respectively, of the filter $h(t)$. *Hint:* Recall from class that

$$H_I(f) = \frac{H_Z(f) + H_Z^*(-f)}{2}$$

$$H_Q(f) = \frac{H_Z(f) - H_Z^*(-f)}{2j},$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

- [10] (b) Draw a circuit that takes as input $x_I(t)$ and $x_Q(t)$ and outputs $y(t)$, while employing only summers, multipliers, oscillators, and *lowpass* filters. (*Note:* A “lowpass filter” is defined for this problem as one whose frequency response is non-zero only for $|f| \leq 5KHz$.)

- [8] (c) Find the output $y(t)$ of your transmitter when $x(t) = 6 \cos(2\pi 1,002,500t + \frac{\pi}{2})$.

- [7] (d) Suppose that $X(f) = H(f)$. Determine whether $x(t)$, the inverse Fourier transform of $X(f)$, could be the output of a DSB-SC system; that is, of the form

$$x(t) = A_c m(t) \cos(2\pi f_c t + \theta)$$

4. Consider the (upper) single-sideband (SSB) signal as derived in class given by:

$$x(t) = A_2 m(t) \cos(2\pi f_c t) - A_2 \hat{m}(t) \sin(2\pi f_c t)$$

- [8] (a) Suppose that $A_2 = 2$ and $m(t) = 5 \text{sinc}^2(50t)$. Sketch $X(f)$.

- [7] (b) From class, recall that the receiver can extract $m(t)$ from $x(t)$ by multiplying by $\cos(2\pi f_c t)$ and lowpass filtering the result. Show that this is true.