ECE 563 - Intro to Comm. and Sig. Proc., Fall 2005

Midterm Exam #1 Make-Up

Thursday, October 20th, 6:30-8:30, ELAB 306

Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\begin{aligned}\cos(\theta) &= \tfrac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)\\ \sin(\theta) &= \tfrac{1}{2j}\left(e^{j\theta}-e^{-j\theta}\right)\end{aligned}$$

$$\begin{aligned}\sin(a\pm b) &= \sin(a)\cos(b)\pm\cos(a)\sin(b)\\ \cos(a\pm b) &= \cos(a)\cos(b)\mp\sin(a)\sin(b)\end{aligned}$$

$$\begin{aligned}\cos(a)\cos(b) &= \tfrac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]\\ \sin(a)\sin(b) &= \tfrac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]\\ \sin(a)\cos(b) &= \tfrac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]\end{aligned}$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t)=\begin{cases}1 & \lvert t\rvert\le 1/2\\ 0 & \lvert t\rvert > 1/2\end{cases}$	$P(f)=\frac{\sin(\pi f)}{\pi f}=\text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0)-\frac{1}{2j}\delta(f+f_0)$
$\Lambda(t)=\begin{cases}1-\lvert t\rvert & \lvert t\rvert\le 1\\ 0 & \lvert t\rvert > 1\end{cases}$	$X(f)=\text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt=\int_{-\infty}^{\infty}\lvert X(f)\rvert^2df$$

1. You desire to send the message signal

$$m(t) = 4\cos(2\pi 5000 t)$$

with an amplitude modulation (AM) system.

[10] (a) Suppose that you choose to modulate your message onto a carrier signal $A_c \cos(2\pi 10^6 t)$ of power 8 (i.e. if the message were zero, the power of the transmitted signal would be 8), and that you want to employ a modulation index of 0.75. Write an expression for the transmitted signal $x(t)$, and roughly sketch $x(t)$.

[5] (b) Provide the simplest possible receiver that can be used to recover $m(t)$. Be sure to provide all necessary parameters (e.g. filter bandwidths, oscillator frequencies, critical component values, etc.).

[12] (c) Disaster strikes! After you have already built your transmitter from (a) and your receiver from (b), you find out that your system is supposed to operate instead with carrier frequency 2×10^6 Hz rather than 10^6 Hz. However, you do not have any more oscillators. Instead, all you have is a square-law device $y(t) = x^2(t)$ and money to buy a single linear time-invariant (LTI) filter (of your choice). Can you take your transmitter from (a), possibly adjust the choice of a, and *run the output of the transmitter from (a)* through these two additional components to make a transmitter that works at $f_c = 2 \times 10^6$ Hz with a convential AM receiver?

[3] (d) Suppose in part (c) that you were able to build a transmitter which perfectly accomplished its goal: doubling the carrier frequency to 2×10^6 Hz to form a signal that can be received with a conventional AM receiver with no additional distortion to your message. What changes are required for the receiver in (b) to receive this signal? Because you desire to save money (and you already have the receiver in (b)), make as few changes as possible.

2. Let

$$z(t) = 5 \sum_{k=-\infty}^{\infty} \Lambda\left(200\left(t - \frac{k}{100}\right)\right)$$

where $\Lambda(x)$ is the triangle function defined in class (height 1 and width 2 centered at $x = 0$). It is easy to observe that $z(t)$ is just a scaled version of the standard triangle repeated infinitely in each direction (draw $5\Lambda(200t)$ and then repeat it every $\frac{1}{100}$ seconds in each direction).

I form the signal $x(t) = 4\cos(2\pi 10^6 t + 2\pi z(t))$.

[10] (a) Find the power, maximum frequency deviation, and bandwidth of $x(t)$. (To do so, you will need to estimate the bandwidth of $\frac{dz(t)}{dt}$. Do not spend too much time doing this - any *minimally justified* answer is fine for the bandwidth of $\frac{dz(t)}{dt}$.)

[5] (b) *Roughly* sketch a portion of $x(t)$ (any sketch that conveys the general idea of how the frequency varies is fine).

[5] (c) Find $Z(f)$, the Fourier transform of $z(t)$. [*Warning: You might find this part difficult! Do it last.*]

3. [10] Suppose that the message signal $A_m \sin(2\pi f_m t)$, where $f_m = 2$ KHz, is input to an FM system with carrier frequency $f_c \gg f_m$ but unknown frequency deviation constant k_f. Suppose that I increase the amplitude A_m until the spectral line located at f_c disappears, and I note that this occurs at $A_m = 3$ volts. Find the frequency deviation constant k_f (in Hz/V).

4. We are designing an analog quadrature amplitude modulation (QAM) system to carry two messages: $m_1(t) = \text{sinc}(10t)\cos(2\pi 100t)$ and $m_2(t) = \text{sinc}(20t)\cos(2\pi 100t)$.

[18] (a) The QAM signal is formed as:

$$x(t) = m_1(t)\cos(2\pi 10^6 t) + m_2(t)\sin(2\pi 10^6 t)$$

and filtered with a bandpass filter $h(t) = \text{sinc}(200t)\sin(2\pi 10^6 t)$. Find the output $y(t) = h(t) * x(t)$ of the transmitter.

[7] (b) Redesign the transmitter from part (a) such that the use of an RF bandpass filter is avoided. Be sure to give all oscillator frequencies and filter impulse responses. Your system should have $m_1(t)$ and $m_2(t)$ as input and $y(t)$ as output.

5. Suppose that the message $A_m \cos(2\pi f_m t)$ is to be transmitted on a unit-amplitude carrier $\cos(2\pi f_c t)$, and we are trying to decide whether to use DSB-SC or conventional AM. A DSB-SC system would simply employ $x(t) = A_m \cos(2\pi f_m t)\cos(2\pi f_c t)$, but, of course, the conventional AM system is more power intensive as it will need to add a carrier $A_c \cos(2\pi f_c t)$ to this DSB-SC system.

   [7] (a) Find the *minimum* percentage increase in transmitted power when conventional AM is employed to convey this message rather than DSB-SC.

   [8] (b) Often the FCC constrains not the average transmitted power but instead the peak transmitted power P_{max}:

$$P_{max} = \frac{\max_t (x_I^2(t) + x_Q^2(t))}{2}$$

   where $x_I(t)$ and $x_Q(t)$ are the in-phase and quadrature components of the transmitted signal, respectively. Find the *minimum* percentage increase in peak transmitted power when conventional AM is employed to convey this message rather than DSB-SC.