ECE 563 - Intro to Comm. and Sig. Proc., Fall 2005

Midterm Exam #1

Tuesday, October 18th, 6:30-8:30, AEBN 119

Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

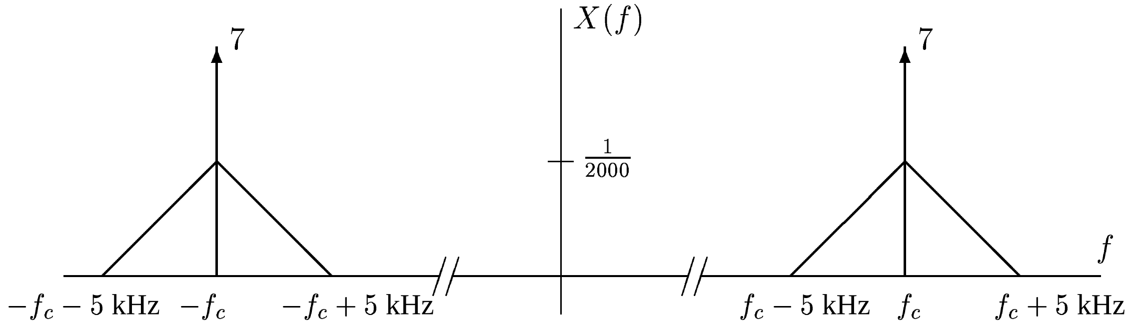
$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right) e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \le 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \le 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

<u>Parseval's Relation:</u> If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. An amplitude modulation (AM) transmitter outputs a signal $x(t)$ with Fourier transform $X(f)$ given by:



The carrier frequency is $f_c = 1$ MHz. You also know that the message spectrum $M(f)$ contains no $\delta(\cdot)$ functions.

[10] (a) Find $x(t)$ and then *roughly* sketch it. [In your drawing, focus on conveying the key points rather than artistic excellence.]

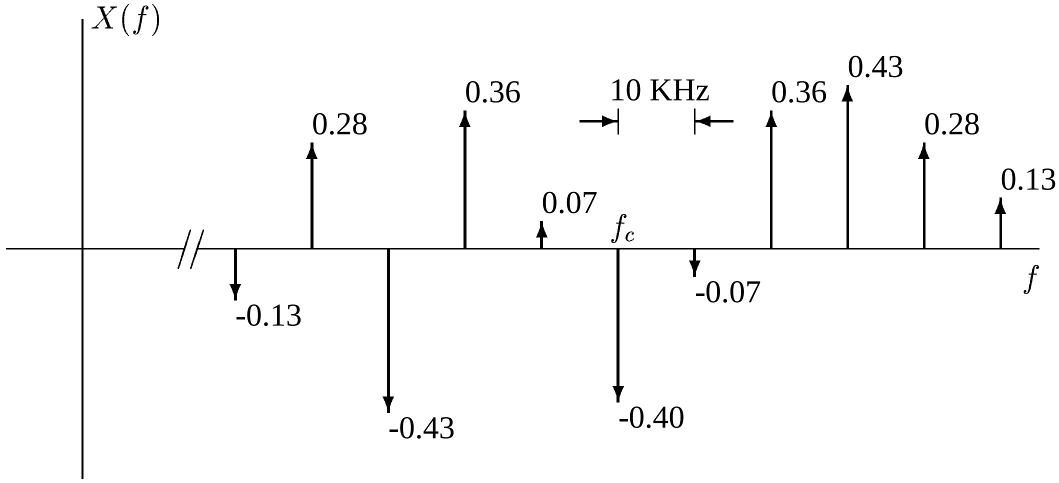
[5] (b) Find the bandwidth of the signal $x(t)$.

[5] (c) Provide the simplest possible receiver that can be used to recover $m(t)$. Be sure to provide all necessary parameters (e.g. filter bandwidths, oscillator frequencies, critical component values, etc.).

[5] (d) *For this specific message*, what is the minimum height to which the delta functions in the figure above can be reduced while still allowing your receiver from (c) to recover $m(t)$?

[5] (e) Suppose I remove the unmodulated carrier component from $x(t)$, leaving just a term of the form $A_2 m(t) \cos(2\pi f_c t)$. Find the energy in the resulting signal.

2. A frequency modulation (FM) transmitter with carrier frequency $f_c = 100$ MHz and carrier amplitude $A_c = 2$ generates a signal $x(t)$ with the following spectrum for $f \geq 0$:



Note that the spectrum consists of Dirac delta functions spaced at 10 KHz. Assume that you know that the message signal $m(t)$ is a sinusoid of the form $m(t) = \cos(2\pi f_m t)$.

[5] (a) Draw the spectrum of $X(f)$ for $f < 0$. [*Note: The picture does not have to be extremely accurate, but make sure I know exactly what you mean.*]

[5] (b) Find f_m, the frequency of the message signal.

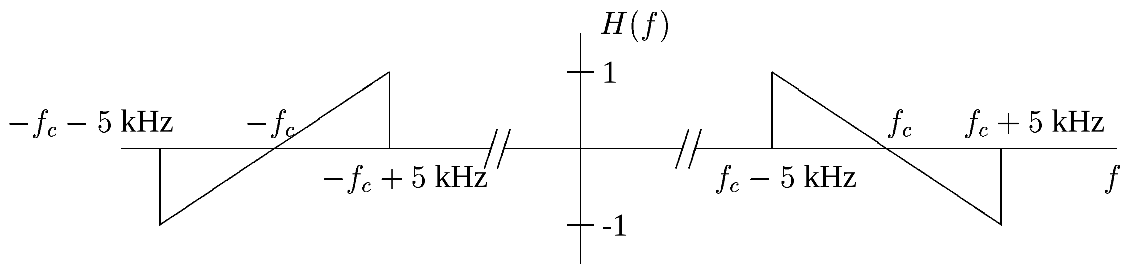
[10] (c) Find β, the modulation index of the FM signal $x(t)$. [*Hint: Recall that the coefficients of the Fourier Series representation for the complex baseband representation of* $x(t)$ *are given by* $z_k = A_c J_k(\beta)$.]

[5] (d) Use β to determine the the maximum frequency deviation $(\Delta f)_{max}$ and bandwidth of $x(t)$.

3. In your first job after graduation from UMass, you are asked to design a transmitter that takes as input two messages ($m_1(t)$ and $m_2(t)$) and generates the output $y(t) = h(t) * x(t)$, where:

$$x(t) = m_1(t)\cos(2\pi f_c t + \phi) + m_2(t)\sin(2\pi f_c t + \phi),$$

where ϕ is some (known) fixed phase offset chosen by your boss, and $h(t)$ has Fourier transform $H(f)$ given by:



[6] (a) Find the in-phase ($x_I(t)$) and quadrature ($x_Q(t)$) components of $x(t)$ in terms of $m_1(t)$, $m_2(t)$ and ϕ.

[8] (b) Find the frequency domain expressions $H_I(f)$ and $H_Q(f)$ for the in-phase and quadrature components, respectively, of the bandpass filter $h(t)$. *Hint:* Recall from class that

$$H_I(f) = \frac{H_Z(f) + H_Z^*(-f)}{2}$$
$$H_Q(f) = \frac{H_Z(f) - H_Z^*(-f)}{2j},$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

[6] (c) Draw a block diagram of a system with inputs $m_1(t)$ and $m_2(t)$, and output $y(t)$ that uses only summers, multipliers, oscillators, and *lowpass* filters. Remember that all signals and filter impulse responses must be real, of course. [*This is an engineering course. Do not include useless extra components!*]

4. [10] Suppose that a real bandpass signal $x(t)$ has quadrature component that is zero; that is, $x_Q(t) = 0$. What does this tell you about the Fourier transform of $X(f)$? (say, versus a signal for which $x_Q(t) \neq 0$).

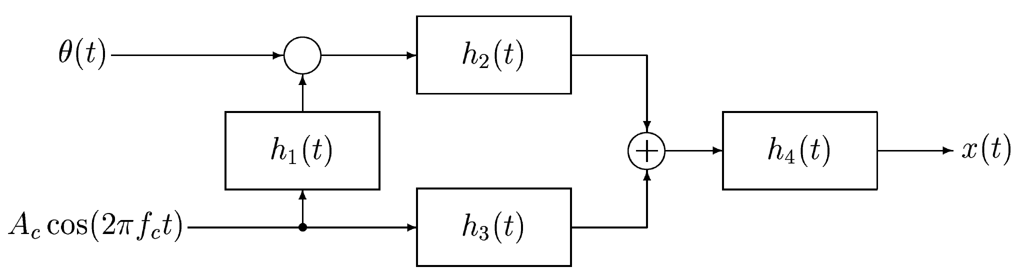
5. We know that the general form of frequency modulation (FM) is given by:

$$x(t) = A_c \cos(2\pi f_c t + \theta(t))$$

where $\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$. If $|\theta(t)| \ll 1$, we showed in class that we can get the expression for narrowband FM as:

$$x(t) = A_c \cos(2\pi f_c t) - A_c\theta(t)\sin(2\pi f_c t)$$

We desire to implement each of these transmitters with this relatively simple circuit:



where $h_1(t)$, $h_2(t)$, $h_3(t)$, and $h_4(t)$ are linear time-invariant (LTI) filters of our choice.

[7] (a) Are there four LTI filters that allow the circuit given above to generate narrowband FM? If so, specify each of the filters (give either $h_i(t)$ **or** $H_i(f)$ for each filter). If not, indicate why it cannot be done.

[8] (a) Are there four LTI filters that allow the circuit given above to generate wideband (or the general form of) FM? If so, specify each of the filters (give either $h_i(t)$ **or** $H_i(f)$ for each filter). If not, indicate why it cannot be done.