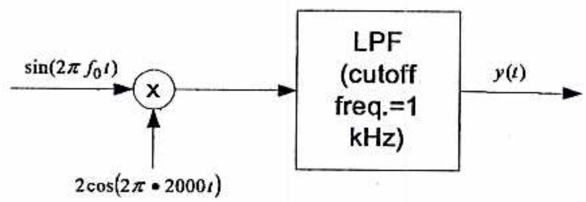
ECE 563 - Fall 2004

Exam 1 (2 hours, 30 minutes; Closed Book - One Sheet of Notes Allowed)

1. *(11%)* Consider the system:



(a) *(6%)* Find two different, **positive** values of f_0 that will generate an output in the form $y(t) = \sin(2\pi \cdot 500t + \theta)$ where θ is some phase shift.

(b) *(5%)* Find θ for each of the values of f_0 found in part (a).

2. *(14%)* Find the power and bandwidth for each of the following two transmitted signals:

(a) *(6%)* $x_1(t) = 10[2 + \sin(200\pi t)]\cos(2\pi \times 10^5 t)$.

(b) *(8%)* $x_2(t) = 10\cos(2\pi \times 10^5 t + 10\sin[200\pi t])$.

3. *(25%)* Let $x(t)$ and $h(t)$ be bandpass signals.

(a) *(8%)* Suppose that the complex envelope of $x(t)$ has Fourier Transform

$$X_l(f) = j\delta(f - 50) + \delta(f + 50).$$

Find the in-phase and quadrature components of $x(t)$.

(b) *(9%)* Suppose that the complex envelope of $h(t)$ has Fourier Transform

$$H_l(f) = -j,\ 0 \le f \le 100$$
$$= 0, \text{ otherwise.}$$

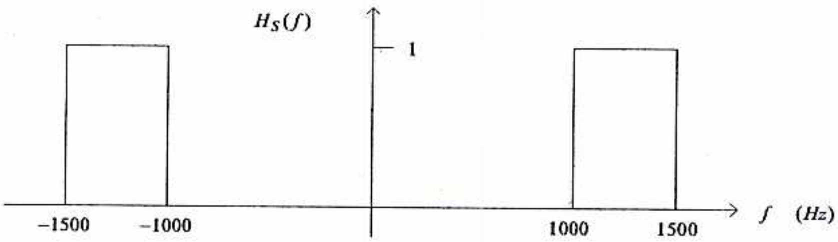
Find the in-phase and quadrature components of $h(t)$.

(c) *(8%)* Now let $y(t) = x(t) * h(t)$ (with $x(t)$ as given in part (a) and $h(t)$ as given in part (b)). Find the in-phase and quadrature components of $y(t)$.

4. *(25%)* Consider the message signal $m(t) = \cos(200\pi t) + \cos(800\pi t)$. We form the DSB-SC signal

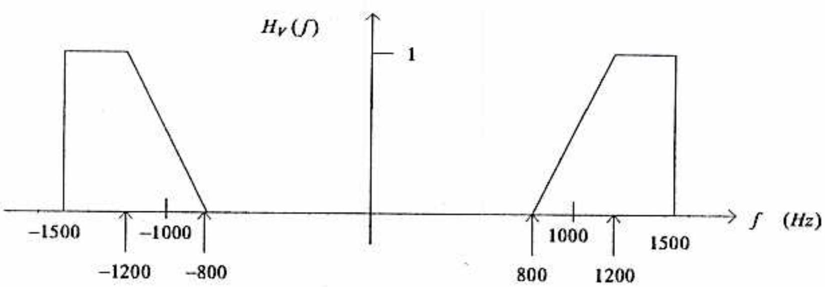
$$x_{DSB}(t) = 2m(t)\cos(2000\pi t).$$

(a) *(9%)* Suppose first that we generate a USSB transmitted signal by putting $x_{DSB}(t)$ through a BPF having the transfer function $H_S(f)$ shown below:



Let $x_{SSB}(t)$ be the filter output. Find the quadrature component (defined with respect to a center frequency of 1 kHz) of $x_{SSB}(t)$. What is the quadrature component's power?

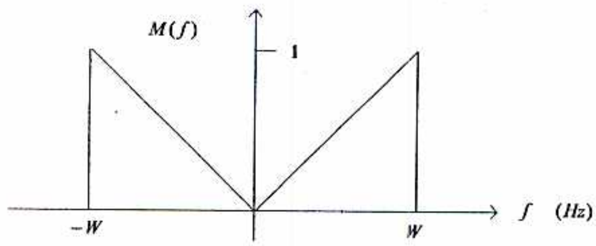
(b) *(9%)* Now suppose that we generate a VSB transmitted signal by putting $x_{DSB}(t)$ through a BPF having the transfer function $H_V(f)$ shown below:



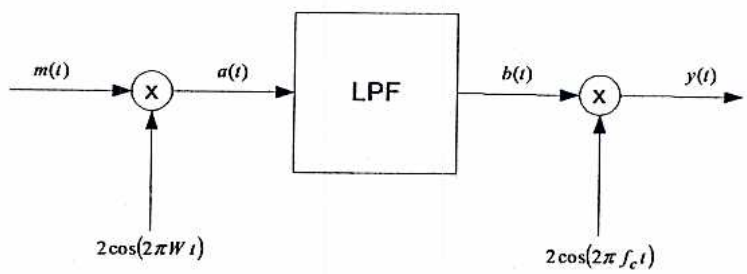
Let $x_{VSB}(t)$ be the filter output. Find the quadrature component (again, defined with respect to a center frequency of 1 kHz) of $x_{VSB}(t)$. What is the quadrature component's power?

(c) *(7%)* Finally, suppose that we want to recover $m(t)$ by putting the transmitted signal through a coherent demodulator. If there is a phase error in the demodulator's local oscillator, with which transmitted signal, $x_{SSB}(t)$ or $x_{VSB}(t)$, would you expect the recovered signal's quality to be worse? To receive credit, you must provide an explanation for your answer. (*Hint:* Consider your answers to parts (a) and (b).)

5. *(25%)* Suppose that $m(t)$ is a lowpass message signal having the Fourier Transform shown below:

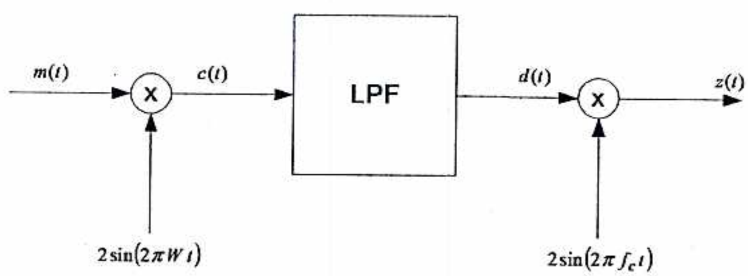


(a) *(9%)* Consider the following system:



where the LPF has a cutoff frequency of W Hz and $f_c \gg W$. Sketch the Fourier Transform of the output signal $y(t)$.

(b) *(10%)* Now consider the following system:



where the LPF cutoff frequency and f_c are the same as in part (a). Sketch the Fourier Transform of the output signal $z(t)$.

(c) *(6%)* Suppose we form the transmitted signal $x(t) = y(t) + z(t)$. What type of transmitted signal is $x(t)$?

Useful Formulas: Trig Identities:

$\cos(a)\cos(b) = \frac{1}{2}\{\cos(a+b)+\cos(a-b)\}$

$\sin(a)\sin(b) = \frac{1}{2}\{\cos(a-b)-\cos(a+b)\}$

$\sin(a)\cos(b) = \frac{1}{2}\{\sin(a+b)+\sin(a-b)\}$

$\cos(a+b) = \cos(a)\cos(b)-\sin(a)\sin(b)$

$\sin(a+b) = \sin(a)\cos(b)+\cos(a)\sin(b)$

$\cos^2(a) = \frac{1}{2}\{1+\cos(2a)\}$

$\sin^2(a) = \frac{1}{2}\{1-\cos(2a)\}$

Fourier Transform Pairs:

sinc(t) $\left(= \frac{\sin(\pi t)}{\pi t}\right) \Leftrightarrow p(f)$ $\left(= 1, |f| < \frac{1}{2}; \ = 0, \text{otherwise}\right)$

$p(t) \Leftrightarrow$ sinc(f)

sinc$^2(t) \Leftrightarrow T(f)$ $\left(= 1-|f|, |f| < 1; \ = 0, \textit{otherwise}\right)$

$T(t) \Leftrightarrow$ sinc$^2(f)$

$\delta(t-s) \Leftrightarrow e^{-j2\pi fs}$

$e^{j2\pi f_0 t} \Leftrightarrow \delta(f-f_0)$

$\cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}\{\delta(f-f_0)+\delta(f+f_0)\}$

$\sin(2\pi f_0 t) \Leftrightarrow \frac{1}{2j}\{\delta(f-f_0)-\delta(f+f_0)\}$

Fourier Transform Properties:

Parseval's Relation: $\int_{-\infty}^{\infty}|x(t)|^2 dt = \int_{-\infty}^{\infty}|X(f)|^2 df$

Time shift: $x(t-s) \Leftrightarrow X(f)e^{-j2\pi fs}$

Frequency shift: $x(t)e^{j2\pi gt} \Leftrightarrow X(f-g)$

Time and frequency scaling: $x(ct) \Leftrightarrow \frac{1}{|c|}X(\frac{f}{c})$

Convolution: $x(t) * y(t) \Leftrightarrow X(f)Y(f)$; $\quad x(t)y(t) \Leftrightarrow X(f) * Y(f)$