ECE 563 - Intro to Comm. and Sig. Proc., Fall 2001

Midterm Exam #1

Wednesday, October 17th, 6:00-8:00, GOES 20

Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \le 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \le 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

<u>Note:</u> The word **estimate** does not mean "guess". In some cases there are multiple acceptable answers, but your answer must be well-justified.

1. A *bandpass* signal $x(t)$ is shown in Figure 1. You can assume that it continues infinitely in each directly with the same behavior shown in $1 \leq t \leq 2$.

   [5] (a) Is $x(t)$ an AM or an FM signal?

   [5] (b) Estimate the power in the signal $x(t)$.

   [10] (c) Estimate the modulation index and bandwidth of the signal $x(t)$.

2. A *bandpass* signal $y(t)$ is shown in Figure 2. You can assume that the trend you see between -1 and 1 continues in each direction.

   [5] (a) Is $y(t)$ an AM or an FM signal?

   [5] (b) Estimate the energy in the signal $y(t)$.

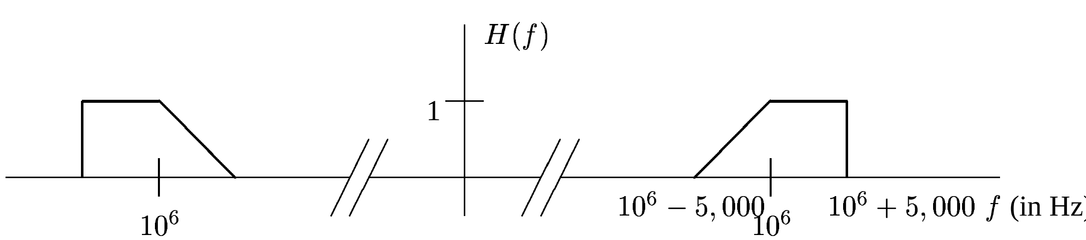
   [10] (c) Estimate the modulation index and bandwidth of the signal $y(t)$.

3. In your first job after graduation from UMass, you are assigned to design a transmitter that outputs $y(t) = h(t) * x(t)$, where

$$x(t) = x_I(t)\cos(2\pi 10^6 t) - x_Q(t)\sin(2\pi 10^6 t)$$

   is a bandpass signal with lowpass signals $x_I(t)$ and $x_Q(t)$, (each of bandwidth 5 KHz) as its in-phase and quadrature components, respectively, and $h(t)$ is a bandpass filter. The inputs to your transmitter are $x_I(t)$ and $x_Q(t)$, and the output is $y(t)$.

   [10] (a) Suppose that the (real) bandpass filter response is as given below, where $H(f)$ is the Fourier transform of $h(t)$:



Sketch the Fourier transforms $H_I(f)$ and $H_Q(f)$ of the in-phase part $h_I(t)$ and quadrature part $h_Q(t)$, respectively, of the filter $h(t)$. *Hint:* Recall from class that

$$\begin{aligned} H_I(f) &= \frac{H_Z(f) + H_Z^*(-f)}{2} \\ H_Q(f) &= \frac{H_Z(f) - H_Z^*(-f)}{2j}, \end{aligned}$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

[5] (b) Find $h_I(t)$ *or* $h_Q(t)$ (your choice!).

[10] (c) Draw a circuit that takes as input $x_I(t)$ and $x_Q(t)$ and outputs $x(t)$, while employing only summers, multipliers, oscillators, and *lowpass* filters.(*Note:* A "lowpass filter" is defined for this problem as one whose frequency response is non-zero only for $|f| \leq 5KHz$.

[5] (d) Find the output $y(t)$ of your transmitter when $x(t) = 3\cos(2\pi 997,500t + \frac{\pi}{4})$.

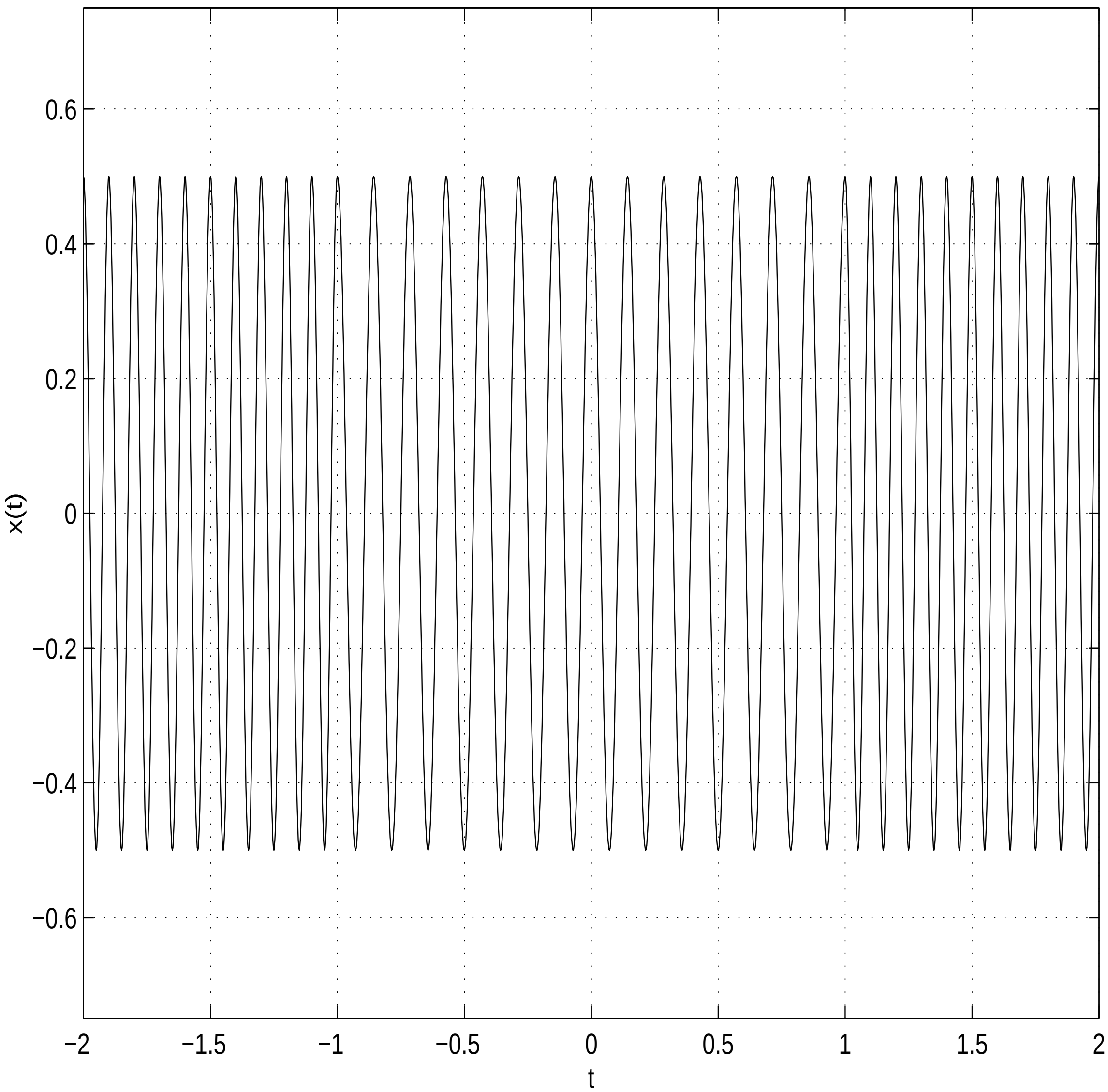


Figure 1: Signal $x(t)$ for Problem 1

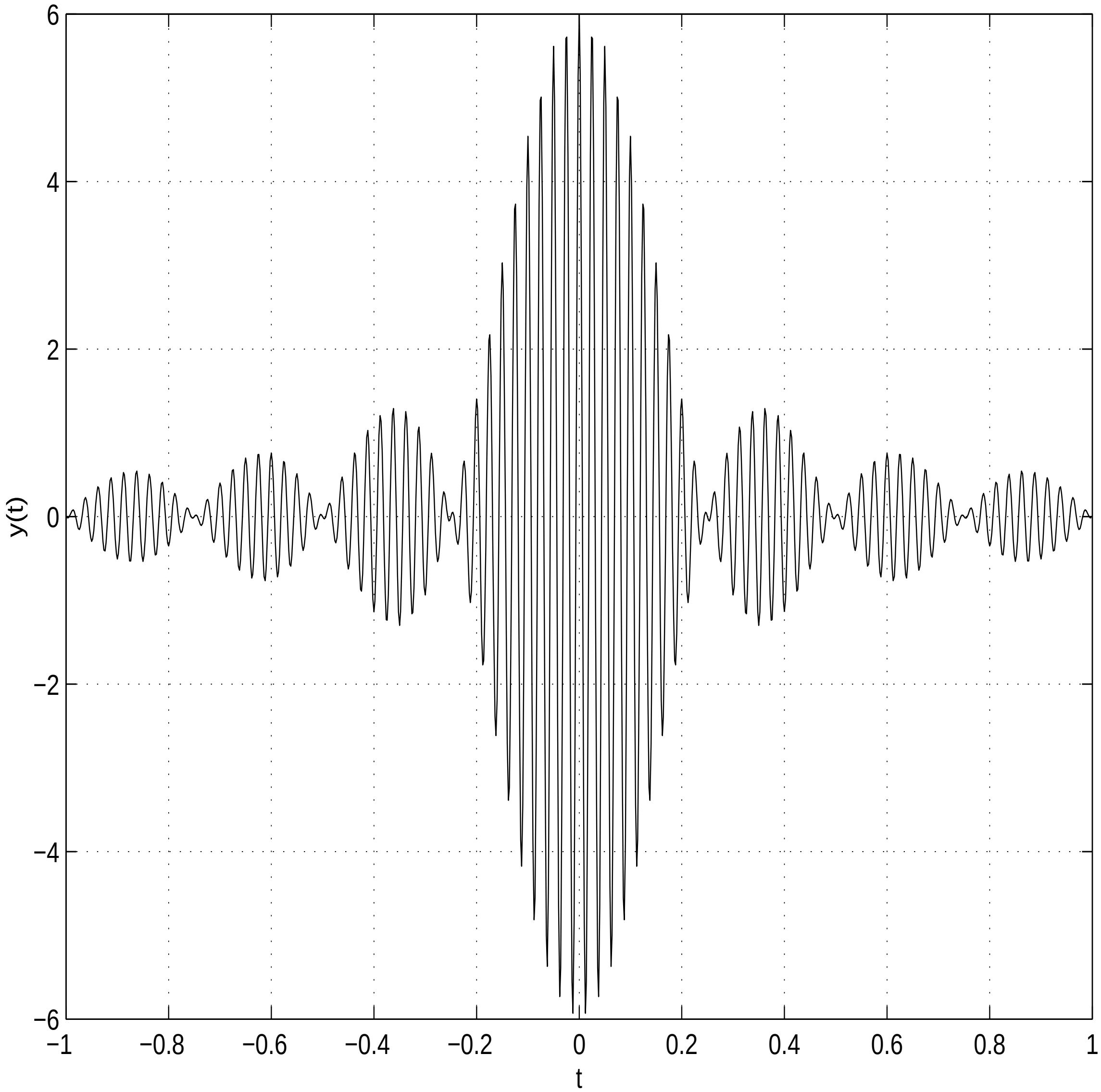


Figure 2: Signal $y(t)$ for Problem 2