ECE 566 - Communication Systems I, Fall 2000

Midterm Exam #1

Thursday, October 19th, 6:00-8:00, ELAB 303

Overview

- The exam consists of five problems for 105 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right) e^{j2\pi \frac{b}{a} f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1 - \lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. [10] Briefly describe in your own words the difference between **amplitude modulation** (AM) and **frequency modulation** (FM). (*Note:* Do *not* just simply write equations for each - tell me what they *mean.*) After addressing the basic differences in the definitions of the two modulation types, identify any performance differences that we have talked about in class.

2. We use the message signal $m(t) = 6\ \text{sinc}^2(3t)$ to **amplitude modulate** a carrier of frequency $f_c = 8$ Hz. The modulation index is 0.8, and the unmodulated carrier (i.e. the output signal if we had set $m(t) = 0$) would have power 8.

   [5] (a) Write an expression for the resulting AM signal $x(t)$.

   [5] (b) Draw a *rough* sketch of $x(t)$.

   [7] (c) Find E_m, the energy in the message signal $m(t)$.

   [5] (d) Find P_x, the power in the resulting AM signal $x(t)$. What is the power efficiency of this AM system?

   [8] (e) Find $X(f)$ and *roughly* sketch $|X(f)|$, where $X(f)$ is the Fourier transform of the signal $x(t)$. What is the bandwidth of the signal $x(t)$?

   [5] (f) Now assume the same message signal $m(t)$ as described above, but consider the more realistic carrier frequency $f_c = 10^6$ Hz. Give a simple receiver that could be employed to recover the signal $m(t)$ from $x(t)$. Be sure to specify all *key* component values.

3. [7] (a) Suppose that I **frequency modulate** (FM) a carrier with power 4 and frequency $f_c = 50$ Hz with the message signal:

   $$m(t) = 4\ \text{sinc}(6t)$$

   The frequency deviation constant is $k_f = 0.5$ Hz/volt. Find the maximum frequency deviation of the resulting signal $x_{FM}(t)$ from the carrier frequency f_c, and estimate the 98% bandwidth of $x_{FM}(t)$.

   [8] (b) Suppose that I **phase modulate** (PM) a carrier with power 4 and frequency $f_c = 50$ Hz with the message signal:

   $$m(t) = 4\ \sin(2\pi 2t)$$

   The phase deviation constant is $k_p = 0.5$ volt^{-1}. Find the maximum frequency deviation of the resulting signal $x_{PM}(t)$ from the carrier frequency f_c, and estimate the 98% bandwidth of $x_{PM}(t)$.

4. Suppose that I have a signal $x(t)$ with Fourier transform

$$X(f) = \frac{1}{400} p\left(\frac{f-f_c}{200}\right) + \frac{1}{400} p\left(\frac{f+fc}{200}\right)$$

where $f_c \gg 1000$.

[5] (a) Find real lowpass signals $x_I(t)$ and $x_Q(t)$ such that

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

[15] (b) Now suppose that I filter the signal $x(t)$ with a filter with impulse response $h(t)$, where the Fourier transform of $h(t)$ is given by:

$$H(f) = \begin{cases} 1, & |f| \leq f_c \\ 0, & |f| > f_c \end{cases} .$$

Let the output of the filter be denoted $y(t) = h(t) * x(t)$. Find real lowpass signals $y_I(t)$ and $y_Q(t)$ such that:

$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

[10] (c) Now suppose that $x(t)$ is any bandpass signal with bandwidth equal to that in (a), and suppose that the quadrature component $x_Q(t)$ is known to be zero. Draw a block diagram of a system with input $x(t)$ and output $h(t) * x(t)$, where $h(t)$ is as given in (b), that uses only summers, multipliers, oscillators, and *lowpass* filters (i.e. filters that are lowpass with bandwidth equal to that of the message signal). **Be sure to give the impulse response of all filters - I want exact expressions.** Remember that all signals and filter impulse responses must be real, of course.

5. Consider the AM waveform:

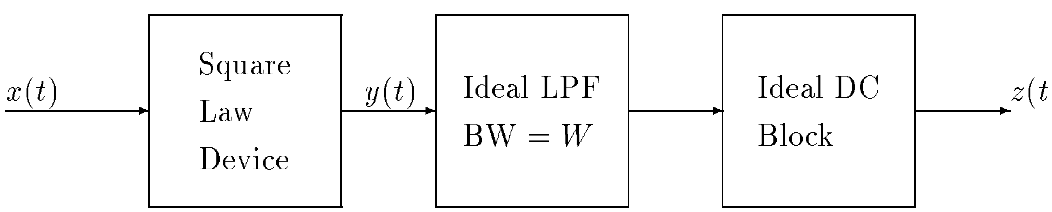
$$x(t) = (A + m(t))\cos(2\pi f_c t).$$

where $m(t)$ has Fourier transform:

$$M(f) = \begin{cases} 1, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}.$$

and it can be assumed that $W \ll f_c$. At the receiver, as shown below,

- We run $x(t)$ through a square-law device to yield $y(t) = x^2(t)$.
- We lowpass filter $y(t)$ with an ideal lowpass filter of bandwidth W.
- We run the output of the lowpass filter through an ideal DC block, which only removes the DC terms (i.e. those exactly at $f = 0$).



The desired output of the receiver is something proportional to $m(t)$; thus, the interfering signal (or "noise") is given by $z(t) - Am(t)$.

[10] (a) Find the energy in the "noise" signal $z(t) - Am(t)$.

[5] (b) Under what conditions on A and $m(t)$ is this receiver effective.