ECE 566 - Communication Systems I, Fall 1999

Midterm Exam #1

October 14th, 6:00-8:00, ELAB 304

Overview

- The exam consists of four problems for 100 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \mathrm{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \mathrm{sinc}^2(f)$

1. The message signal $m(t) = \text{sinc}(\frac{t}{2})$ is used to amplitude modulate the carrier $\cos(2\pi 5t)$.

   [15] (a) Suppose that the system is a double-sideband suppressed carrier (DSB-SC) system; thus, $x(t) = m(t)\cos(2\pi 5t)$.

   - *Roughly* sketch $x(t)$.
   - Find and sketch $X(f)$. What is the bandwidth of $x(t)$?
   - Is $x(t)$ a power or an energy signal (or neither)? If it is a power signal, find its power. If it is an energy signal, find its energy.

   [10] (b) Suppose that the system is a conventional AM system with modulation index a and carrier amplitude $A_1 = 1$; thus, $x(t) = (1 + am_n(t))\cos(2\pi 5t)$, where $m_n(t) = \frac{m(t)}{\max|m(t)|}$ as defined in class.

   - Find and *roughly* sketch $X(f)$ for $a = 0.8$.
   - What is the *maximum* value of a such that $(1+am_n(t))$ is the output of the ideal envelope detector $x_e(t) = \sqrt{x_I^2(t) + x_Q^2(t)}$, when $x(t)$ is the input? (Note that $x_I(t)$ and $x_Q(t)$ denote the in-phase and quadrature components of $x(t)$, respectively).

2. The message signal $m(t)$, defined (in volts) as

$$m(t) = \begin{cases} 1 - |t|, & |t| \leq 2 \\ -1, & \text{otherwise} \end{cases},$$

   is frequency modulated with frequency deviation constant $k_f = 2$ Hz/volt onto a carrier with amplitude 3 volts and frequency $f_c = 4$ Hz.

   [5] (a) Sketch *roughly* the resulting FM signal.

   [10] (b) Find the modulation index and approximate the 98% bandwidth of the resulting FM signal. This will be problematic to do exactly with the formulae given in class, since $m(t)$ is not strictly a lowpass signal. Use your engineering judgment to give suitable answers for this part.

3. In your first job after graduation from UMass, you are assigned to design a transmitter that outputs $y(t) = h(t) * x(t)$, where

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

is a bandpass signal with lowpass signals $x_I(t)$ and $x_Q(t)$ as its in-phase and quadrature components, respectively, and $h(t)$ is a bandpass filter. The inputs to your transmitter are $x_I(t)$ and $x_Q(t)$, and the output is $y(t)$.

[5] (a) Suppose that you were not paying attention in ECE 566. You might form $x(t)$ from $x_I(t)$ and $x_Q(t)$ and then filter it with $h(t)$. Indicate why this is a bad idea.

[10] (b) Suppose that you were paying attention in ECE 566. Draw a circuit that takes as input $x_I(t)$ and $x_Q(t)$ and outputs $y(t)$, while employing only summers, multipliers, oscillators, and *lowpass* filters.

[10] (c) Suppose your carrier frequency is $f_c = 10000$ and that the frequency response of your bandpass filter is given by:

$$H(f) = \begin{cases} 2 & 9500 \le |f| \le 10000 \\ 1 & 10000 \le |f| \le 10500 \\ 0 & \text{otherwise} \end{cases}$$

Find and *roughly* sketch $H_I(f)$ and $H_Q(f)$. *Hint:* Recall from class that:

$$\begin{aligned} H_I(f) &= \frac{H_Z(f) + H_Z^*(-f)}{2} \\ H_Q(f) &= \frac{H_Z(f) - H_Z^*(-f)}{2j} \end{aligned}$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

[10] (d) If $x_I(t) = 2\cos(2\pi 250t)$ and $x_Q(t) = 0$, find the output $y(t)$ if $h(t)$ is the filter described in part (c). (*Note:* There are at least two separate ways to solve this part - one requires the result of part (c) and the other does not. I will give you bonus points if you do it both ways).

4. [10] (a) I have a square-law device with $y(t) = a_0 + a_1 x(t) + a_2 x^2(t)$ as output when $x(t)$ is the input, where a_0, a_1, and a_2 are positive constants. Suppose I also have the capability to design an ideal filter of any sort. Show how I can use the square-law device and linear filtering to build a circuit that doubles the frequency of an input sinusoid of known frequency f_1.

   [15] (b) Design a circuit that accepts two inputs:

$$\cos(2\pi f_0 t) \quad \text{and} \quad \cos(2\pi (f_0 + \Delta f)t + \theta)$$

   and outputs a voltage that is proportional to the frequency difference Δf. Note that f_0 and θ are unknown (as, of course, is Δf), but you can assume that $\Delta f \ll f_0$. You can use any of the standard components from class: multipliers, summers, envelope detectors, filters of any type, etc.