ECE 566 - Communication Systems I, Fall 1998

Midterm Exam #1

October 15th, 6:00-8:00, ELAB 304

Overview

- The exam consists of four problems for 100 points plus 15 bonus points sprinkled throughout. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because . . . " so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Instructions for Problems 1 and 2:

On each of the next two pages is a signal $x(t)$.

For each of the signals:

(a) [5] Identify whether the signal is an AM or FM signal.

(b) **If the signal is an AM signal:**

- [5] Assuming that the message $m(t)$ has no DC component (i.e. its average value is zero), give a possible message signal $m(t)$.
- [5] Assuming that the message $m(t)$ has no DC component (i.e. its average value is zero), give the modulation index and power efficiency of the system.
- [10] Assuming that $x(t)$ continues infinitely in time in both directions with the same pattern, sketch $|X(f)|$ (where $X(f)$ is the Fourier transform of the AM signal). **What is the bandwidth of the signal $x(t)$?**
- [5] Give a detailed circuit using only basic components (resistors, capacitors, inductors, diodes, etc.) with input $x(t)$ and output $m(t)$. Be sure to give important component values.

(c) **If the signal is an FM signal:**

- [7] Give the message signal $m(t)$ if the carrier frequency is 3.5 Hz and the frequency deviation constant is $k_f = 0.5$ Hz/volt. *Hint: The message changes at most once per second.*
- [8] Assuming that the bandwidth of the rectangular pulse $p(t)$ can be approximated as 4 Hz, find the modulation index β.
- [5 BONUS] Use Carson's Rule to estimate the bandwidth of the signal.

Problem 1

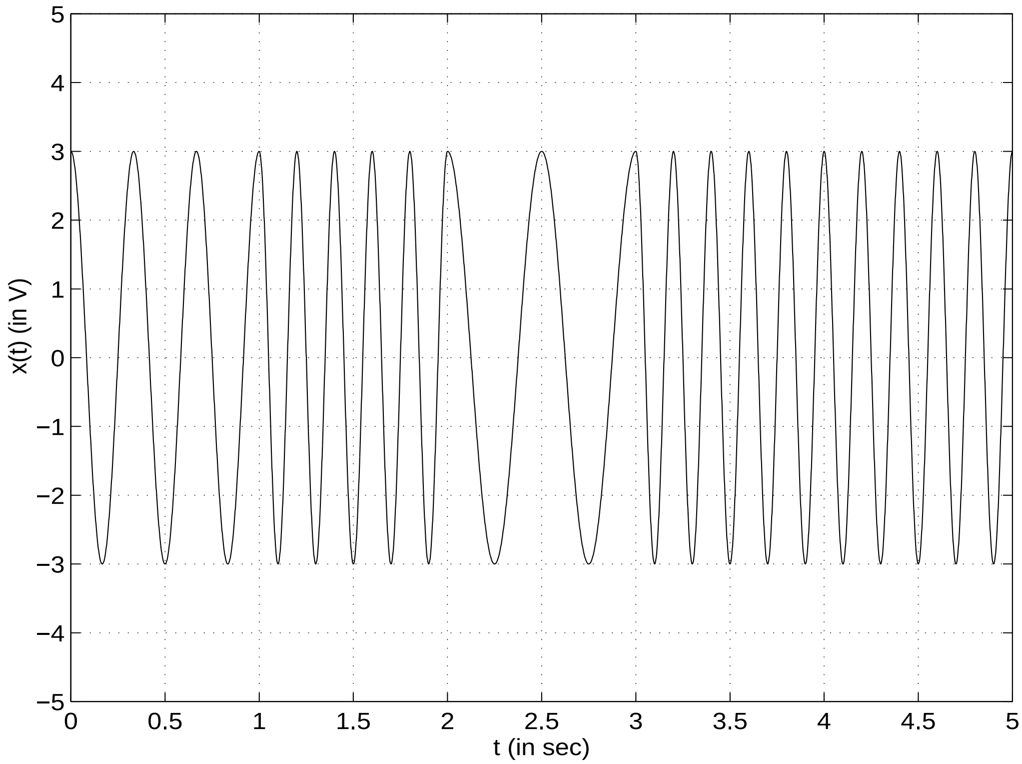

Figure 1: Signal $x(t)$ for Problem 1

Problem 2

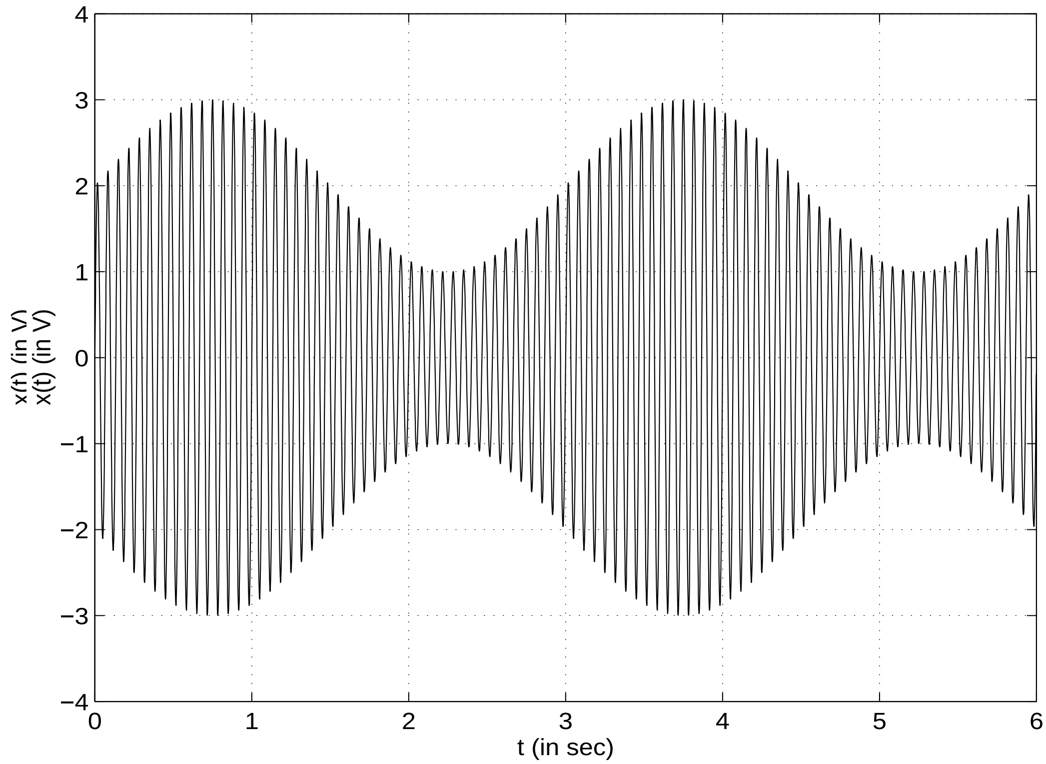

Figure 2: Signal $x(t)$ for Problem 2

3. Suppose that we have a bandpass signal $x(t)$ of bandwidth $2W$ around carrier frequency f_c, which we desire to filter with the following bandpass filter:

$$h(t) = W\text{sinc}(Wt)\cos\left(2\pi f_c t - \frac{\pi}{4}\right) + 2W\text{sinc}(2Wt)\cos\left(2\pi f_c t + \frac{\pi}{4}\right)$$

As we know from class, building *bandpass* filters is hard so we desire instead to do our filtering with *lowpass* filters by completing the following steps.

[10] (a) Find the in-phase ($h_I(t)$) and quadrature ($h_Q(t)$) components of $h(t)$.

[10] (b) Find and *roughly* sketch the frequency responses $H_I(f)$ and $H_Q(f)$.

[10 BONUS] (c) Draw a block diagram of a system with input $x(t)$ and output $h(t) * x(t)$ that uses only summers, multipliers, oscillators, and *lowpass* filters (i.e. filters that are lowpass with bandwidth W). Be sure to give the frequency response of *all* filters. Remember that all signals and filter impulse responses must be real, of course.

[10] (d) Find $y(t) = h(t) * x(t)$ if

$$x(t) = \cos\left(2\pi \frac{3W}{4} t\right)\cos(2\pi f_c t)$$

4. Miscellaneous AM questions:

[10] (a) Suppose I have a signal $x(t) = m(t)\cos(2\pi f_c t)$, where $m(t)$ is a lowpass signal with bandwidth $W \ll f_c$. Find a filter (give $h(t)$ or $H(f)$) for this signal such that the filter output $y(t) = h(t) * x(t)$ is given by:

$$y(t) = m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t$$

where $\hat{m}(t)$ is the Hilbert Transform of $m(t)$.

[10] (b) I have the DSB-SC signal $x(t) = m(t)\cos(2\pi f_1 t)$, where $m(t)$ is a lowpass signal with bandwidth $W \ll f_1$. However, I want to change the carrier frequency to a higher frequency f_c, where $W \ll |f_c - f_1|$. Give a circuit which uses only oscillators, multipliers, summers, and filters that takes $x(t)$ as input and yields $y(t) = m(t)\cos(2\pi f_c t)$ as output.