ECE 563 - Intro to Comm. and Sig. Proc., Fall 2006

Final Exam

Monday, December 18th, 8:00-10:00am, GOES 20

Overview

- The exam consists of four problems for 100 points. The points for each part of each problem are given in brackets **-** you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **three page-sides** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write "this must be wrong because . . . " so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\begin{aligned} \cos(\theta) &= \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \\ \sin(\theta) &= \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \end{aligned}$$

$$\begin{aligned} \sin(a \pm b) &= \sin(a)\cos(b) \pm \cos(a)\sin(b) \\ \cos(a \pm b) &= \cos(a)\cos(b) \mp \sin(a)\sin(b) \end{aligned}$$

$$\begin{aligned} \cos(a)\cos(b) &= \tfrac{1}{2}\left[\cos(a-b) + \cos(a+b)\right] \\ \sin(a)\sin(b) &= \tfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \\ \sin(a)\cos(b) &= \tfrac{1}{2}\left[\sin(a-b) + \sin(a+b)\right] \end{aligned}$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right) e^{j2\pi \frac{b}{a} f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1 - \lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

<u>Parseval's Relation:</u> If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. Using an amplitude modulation (AM) system of bandwidth $W = 5$ KHz, I desire to send a sinusoidal message:

$$m(t) = 5\cos(2\pi 3000 t)$$

[7] (a) Because I want to be energy efficient, I first attempt to use DSB-SC and form the transmitted signal:

$$x(t) = 2m(t)\cos(2\pi f_c t)$$

The received signal is $R(t) = x(t) + W(t)$, where $W(t)$ is white noise with power spectral density $S_W(f) = \frac{N_0}{2}$. Find:

- The transmitted power.
- The output signal-to-noise ratio (SNR) of the standard DSB-SC receiver (of bandwidth W - do **not** cut the bandwidth back to that of the sinusoid) in terms of N_0.

[8] (b) Alas, it is hard to build a phase coherent receiver, and thus I decide to employ conventional AM. Thus, I add a carrier component to my system from (a) until the simple conventional AM receiver functions properly (of course, I want to use the smallest such carrier component). Find:

- The transmitted power.
- The output signal-to-noise ratio (SNR) of the conventional AM receiver (of bandwidth W - do **not** cut the bandwidth back to that of the sinusoid) in terms of N_0.

[5] (c) Suppose that the systems from parts (a) and (b) employ the same transmitted power. Which system, (a) or (b), has the better output SNR? How much better is it?

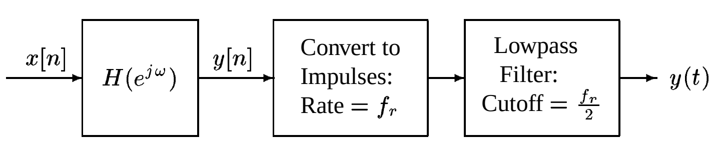
2. Consider the discrete-time signal:

$$x[n] = 5\text{sinc}^2[n/20]$$

[10] (a) Find $X_{DT}\left(e^{j\omega}\right)$, the discrete-time Fourier transform (DTFT) of $x[n]$. (*Note: You do not need a table of DTFT pairs to do this. Instead, think of this as a sampled version of some $x(t)$ at some f_s, and then use $X(f)$ and f_s to find $X_{DT}\left(e^{j\omega}\right)$.*)

[5] (b) Suppose I take a 1024-point discrete Fourier transform (DFT) of $x[n]$. Find (approximately is fine) $\frac{X[35]}{X[0]}$, the ratio of the 36^{th} element of the DFT to the 1^{st} element of the DFT.

[10] (c) Suppose we employ a discrete-time filter $H(e^{j\omega})$ and our standard reconstruction circuitry:



Find $H(e^{j\omega})$ and f_r such that $y(t) = A(\text{sinc}^2(5000t) + \text{sinc}(10000t))$, where A is any amplitude of your choosing (i.e. Do not worry about the amplitude).

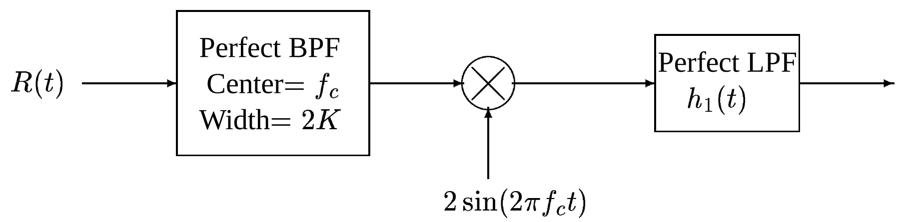
3. Consider the transmitted quadrature amplitude modulation (QAM) signal:

$$x(t) = 10\text{sinc}^2(10t)\cos(2\pi f_c t) + 20\text{sinc}(5t)\sin(2\pi f_c t)$$

[10] (a) Find (in any order):

- The energy in $x_Q(t)$, the quadrature component of $x(t)$.
- The bandwidth of $x(t)$.

[10] (b) Consider the received signal $R(t) = x(t) + W(t)$, where $W(t)$ is white noise with power spectral density $S_W(f) = \frac{N_0}{2}$. Suppose my goal is to extract the quadrature component $20\text{sinc}(5t)$ (i.e. everything other than such is considered "noise"). The natural receiver to extract such is given by:

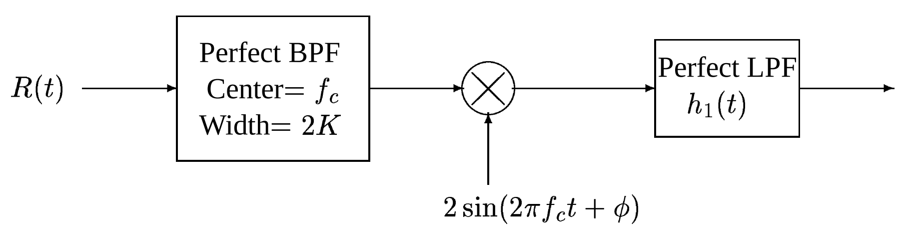


where $h_1(t)$ has frequency response given by:

$$H_1(f) = \begin{cases} 1, & |f| \leq K \\ 0, & \text{otherwise} \end{cases}$$

Find the following three items: (i) K, (ii) the resulting output energy due to the signal, (iii) the resulting output power due to the noise.

[10] (c) Consider the received signal $R(t) = x(t) + W(t)$, where $W(t)$ is white noise with power spectral density $S_W(f) = \frac{N_0}{2}$. Suppose my goal is to extract the quadrature component $20\text{sinc}(5t)$ (i.e. everything other than such is considered "noise"). Unfortunately, I have a small phase error ϕ in my receiver, so that the receiver is actually:

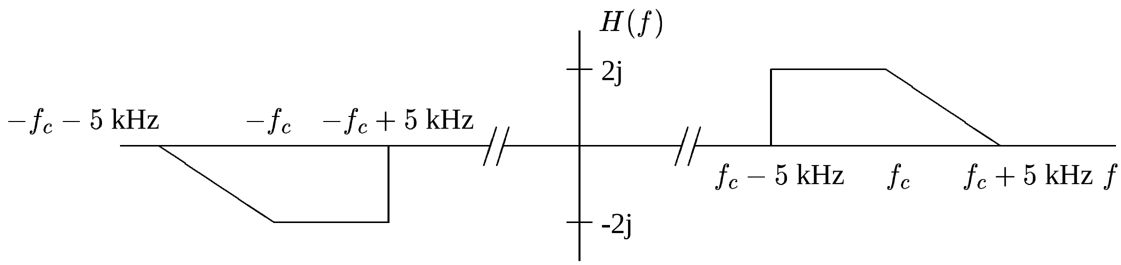


where $h_1(t)$ has frequency response given by:

$$H_1(f) = \begin{cases} 1, & |f| \leq K \\ 0, & \text{otherwise} \end{cases}$$

Characterize the degradation in the system output due to: (i) signal power loss due to the phase mismatch, (ii) "leakage" from the in-phase component into this quadrature branch in terms of E_{x_I}, the energy of the in-phase component.

4. Your new boss asks you to design a bandpass filter to be applied to inputs $y(t)$ of bandwidth 10 kHz around a carrier f_c. The filter response that she wants is given by:



[15] (a) However, your group is not good at designing bandpass filters, so your boss tells you "Convert $y(t)$ to baseband, sample the in-phase and quadrature components, apply digital filtering to those, and then convert it all of the way back up to bandpass." Design such a system, giving all sampling frequencies, frequency responses, etc. to complete this task.

[10] (b) Suppose that input to your filter is a sinusoidal message in noise; that is, $y(t) = A_m \cos(2\pi f_c t) + W(t)$, where $W(t)$ is white noise with power spectral density $S_W(f) = \frac{N_0}{2}$. Find the signal-to-noise ratio (SNR) at the filter output in terms of A_m and N_0. (*Note: If you get an unwieldy integral, feel free not to evaluate such, but be sure all of the limits, etc. are given.*)