

ECE 603 - Probability and Random Processes, Fall 2006

**Homework #5 Supplemental Questions
NOT TO BE TURNED IN!**

1. Let X and Y be random variables with respective probability density functions

$$f_X(x) = \frac{1}{\sqrt{6\pi}} e^{-\frac{(x-3)^2}{6}} \quad \text{for } -\infty < x < \infty$$

$$f_Y(y) = \frac{1}{\sqrt{24\pi}} e^{-\frac{y^2}{24}} \quad \text{for } -\infty < y < \infty$$

(a) Find $P[X \geq 6]$.

(b) Find $E[(X + 2)^2]$.

For parts (c) and (d), suppose that we further know that X and Y are **jointly Gaussian** and that the correlation coefficient between X and Y is $\rho_{X,Y} = \frac{1}{6}$.

(c) Are X and Y independent?

(d) Let $Z = 3X - 2Y$. Find $E[Z]$, $E[Z^2]$, and the probability density function $f_Z(z)$ of Z .

2. Random variable X and Y have the joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} c(x+1) & 0 < x < 1, 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of c .

(b) Find the **marginal** probability density function $f_X(x)$.

(c) Find the **conditional** probability density function $f_{Y|X}(y|x)$.

(d) Find $E[Y|X = x]$ and $\text{Var}[Y|X = x]$.