ECE 563 - Introduction to Communications and Signal Processing, Fall 2005

Final Exam

Saturday, December 17th, 8:00-10:00 a.m., Goesmann 64

Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets **-** you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **three page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because . . . " so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\begin{aligned}\cos(\theta) &= \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)\\ \sin(\theta) &= \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)\end{aligned}$$

$$\begin{aligned}\sin(a \pm b) &= \sin(a)\cos(b) \pm \cos(a)\sin(b)\\ \cos(a \pm b) &= \cos(a)\cos(b) \mp \sin(a)\sin(b)\end{aligned}$$

$$\begin{aligned}\cos(a)\cos(b) &= \tfrac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]\\ \sin(a)\sin(b) &= \tfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]\\ \sin(a)\cos(b) &= \tfrac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]\end{aligned}$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$$

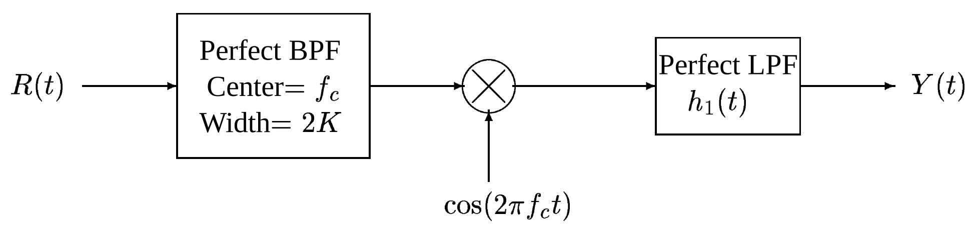
1. Let the message signal $m(t) = \text{sinc}(5t)$ amplitude modulate the carrier $15\cos(2\pi f_c t)$ to form the transmitted signal:

$$x(t) = 15\text{sinc}(5t)\cos(2\pi f_c t)$$

[15] (a) Find (in any order):

- The power in $x(t)$.
- The energy in $x(t)$.
- $X(f)$, the Fourier Transform of $x(t)$.
- A sketch of $X(f)$, the Fourier Transform of $x(t)$.
- The bandwidth of $x(t)$.

The received signal $R(t) = x(t) + W(t)$, where $W(t)$ is a zero-mean Gaussian white noise process with power spectral density $S_W(f) = \frac{N_0}{2}$, is input to the following receiver:



where $h_1(t)$ has frequency response given by:

$$H_1(f) = \begin{cases} 1, & |f| \leq K \\ 0, & \text{otherwise} \end{cases}$$

and K is the bandwidth of $m(t)$.

[5] (b) Characterize the signal portion of $Y(t)$ in the frequency domain (use the frequency-domain tool appropriate for this type of signal). Find the energy in the signal portion of $Y(t)$.

[5] (c) Characterize the noise portion of $Y(t)$ in the frequency domain (use the frequency-domain tool appropriate for this type of signal). Find the power in the noise portion of $Y(t)$.

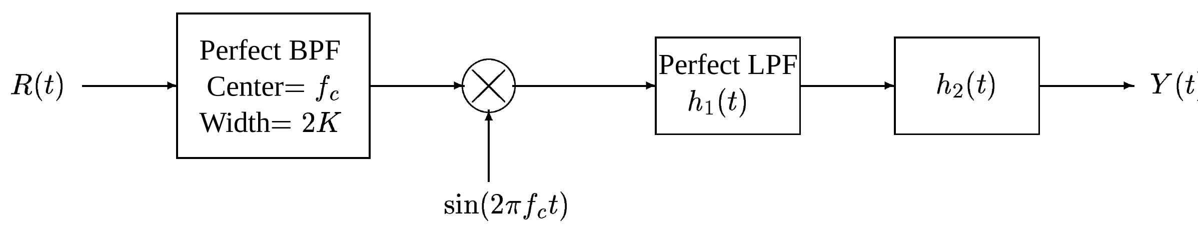
2. The message signal $m(t) = 4\cos(2\pi 10t)$ is transmitted in an upper sideband single-sideband (SSB) system as:

$$x(t) = 3m(t)\cos(2\pi f_c t) - 3\hat{m}(t)\sin(2\pi f_c t)$$

where $\hat{m}(t)$ is the Hilbert transform of $m(t)$; that is, the Fourier Transform of $\hat{m}(t)$ is given by:

$$\hat{M}(f) = \begin{cases} -jM(f), & f \geq 0 \\ jM(f), & f < 0 \end{cases}$$

The received signal $R(t) = x(t) + W(t)$, where $W(t)$ is a zero-mean Gaussian white noise process with power spectral density $S_W(f) = \frac{N_0}{2}$, is input to the following receiver:



where $h_1(t)$ has frequency response given by:

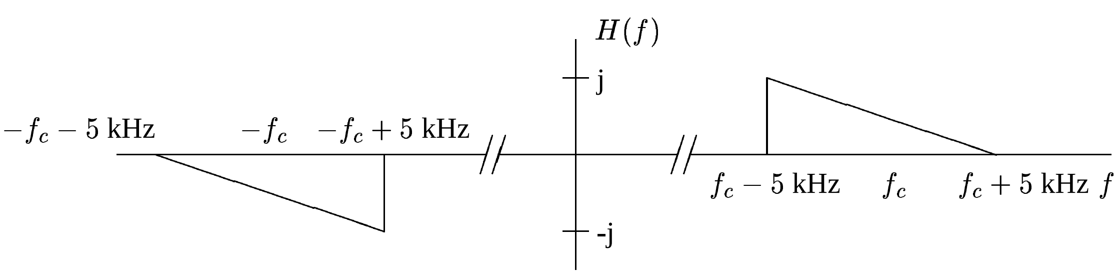
$$H_1(f) = \begin{cases} 1, & |f| \leq K \\ 0, & \text{otherwise} \end{cases}$$

where K is the bandwidth of $m(t)$, and $h_2(t)$ has frequency response:

$$H_2(f) = \begin{cases} -j, & f \geq 0 \\ j, & f < 0 \end{cases}$$

[15] Find the signal-to-noise ratio (SNR) in $Y(t)$.

3. On your first job after leaving UMass, you are asked to design a bandpass filter $H(f)$ to filter some input bandpass signal $x(t)$. The filter $H(f)$ has frequency response:



However, your group is not good at designing bandpass filters, so your boss tells you "Convert $x(t)$ to baseband, sample the in-phase and quadrature components, apply digital filtering to those, and then convert it all of the way back."

[25] Design such a system, giving all sampling frequencies, frequency responses, etc. to complete this task.

4. A random process is defined as:

$$X(t) = Y \cdot t + Z$$

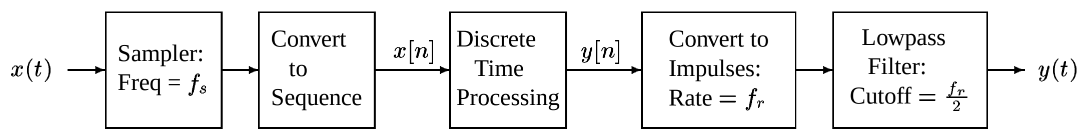
where Y and Z are independent zero-mean jointly-Gaussian random variables, each with variance σ^2.

[10] (a) Find the mean $m_X(t) = E[X(t)]$ and autocorrelation function $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$ of $X(t)$.

[5] (b) Is $X(t)$ wide-sense stationary?

[5] (c) Find $P(X(2) > 3)$, the probability that $X(2)$ is greater than 3.

5. Consider the following system to process continuous-time signals with discrete-time processing (Note: This is the same system we have been using in class, with sampling rate f_s for the C/D and sampling rate f_r for the D/C).



Suppose that the input is given by $x(t) = 50\text{sinc}^2(100t)$ and the sampling rate of the C/D is given by $f_s = 300$ Hz. The discrete-time processing consists of a decimation by a factor of two (which consists of a lowpass filter followed by a downsampler) and then an interpolation by a factor of three (which consists of a "zero insertion" followed by a lowpass filter). The sampling rate of the D/C is given by $f_r = 450$ Hz.

[15] Sketch the spectrum at the output of each of the blocks (do not worry about amplitudes - just give the shape), including those in the discrete-time processing, and use the resulting $Y(f)$ to find $y(t)$.