ECE 563 - Introduction to Communications and Signal Processing, Fall 2001

Final Exam

Monday, December 17th, 8:00-10:00 a.m., ELAB 303

Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets **-** you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **three page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\begin{aligned}\cos(\theta) &= \tfrac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)\\ \sin(\theta) &= \tfrac{1}{2j}\left(e^{j\theta}-e^{-j\theta}\right)\end{aligned}$$

$$\begin{aligned}\sin(a\pm b) &= \sin(a)\cos(b)\pm\cos(a)\sin(b)\\ \cos(a\pm b) &= \cos(a)\cos(b)\mp\sin(a)\sin(b)\end{aligned}$$

$$\begin{aligned}\cos(a)\cos(b) &= \tfrac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]\\ \sin(a)\sin(b) &= \tfrac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]\\ \sin(a)\cos(b) &= \tfrac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]\end{aligned}$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t)=\begin{cases}1 & \lvert t\rvert\le 1/2\\ 0 & \lvert t\rvert > 1/2\end{cases}$	$P(f)=\frac{\sin(\pi f)}{\pi f}=\text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0)-\frac{1}{2j}\delta(f+f_0)$
$x(t)=\begin{cases}1-\lvert t\rvert & \lvert t\rvert\le 1\\ 0 & \lvert t\rvert > 1\end{cases}$	$X(f)=\text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty}|x(t)|^2dt=\int_{-\infty}^{\infty}|X(f)|^2df$$

1. [15] Give a description of the positive and negative aspects of each of:

   - Double-sideband suppressed carrier (DSB-SC)
   - Conventional amplitude modulation (AM)
   - Frequency modulation (FM)

   for broadcast radio applications. Compare and contrast their performance with regard to basic metrics such as required transmission power, bandwidth, and receiver complexity. Also indicate the relative importance of these basic metrics for broadcast radio (For example: Do you care more about transmission power or receiver complexity for broadcast radio?).

2. Suppose that you are interested in analyzing the spectrum of the DSB-SC signal:

$$X(t) = M(t)\cos(2\pi 10^6 t + \Theta)$$

   where $M(t)$ is a zero-mean, wide-sense stationary (WSS) Gaussian random process with autocorrelation function $R_M(\tau) = 3000 \text{ sinc}^2(3000\tau)$, and Θ is a random variable uniformly distributed between 0 and 2π. Fortunately, you have a colleague who remembers her ECE 314, and she is able to derive the autocorrelation function of $X(t)$ for you in terms of the autocorrelation function of $M(t)$:

$$R_X(\tau) = \frac{R_M(\tau)}{2}\cos(2\pi 10^6 \tau)$$

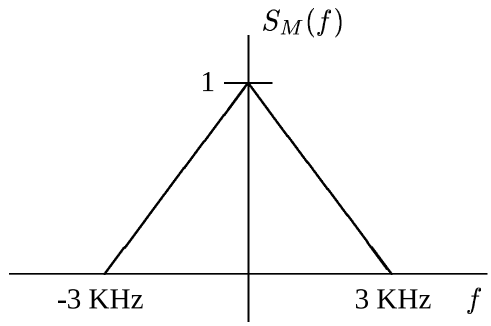
   [5] (a) Find the power in $X(t)$.

   [10] (b) Find and roughly sketch $S_X(f)$, the power spectral density of $X(t)$. **Be sure to show all of your work, assuming only the basic properties of the Fourier Transform.**

3. Suppose that the received waveform in a communication system is given by

$$X(t) = M(t)\cos(2\pi f_c t + \theta) + N(t)$$

where $M(t)$ is a message, which is modeled as a lowpass ($W = 3$ KHz) zero-mean wide-sense stationary (WSS) Gaussian random process with power spectral density given by $S_M(f)$:



θ is uniformly distributed between 0 and 2π, and the noise process $N(t)$ is modeled as white Gaussian noise with power spectral density $S_N(f) = \frac{N_0}{2}$.

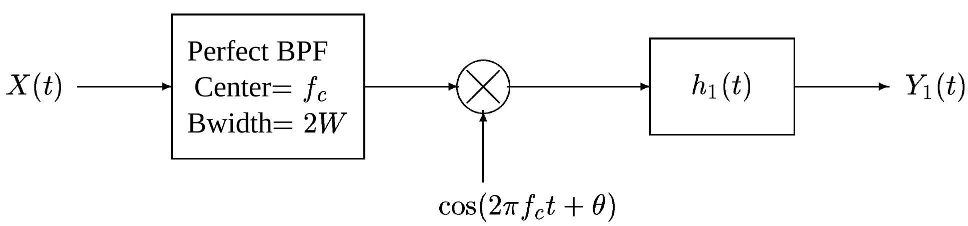
[5] (a) Find the power in the signal part of $X(t)$; that is, find

$$P_S = E[(M(t)\cos(2\pi f_c t + \theta))^2]$$

[5] (b) Find the power in the noise part of $N(t)$; that is, find

$$P_N = E[N^2(t)]$$

[10] (c) Suppose that we process $X(t)$ with the following receiver:



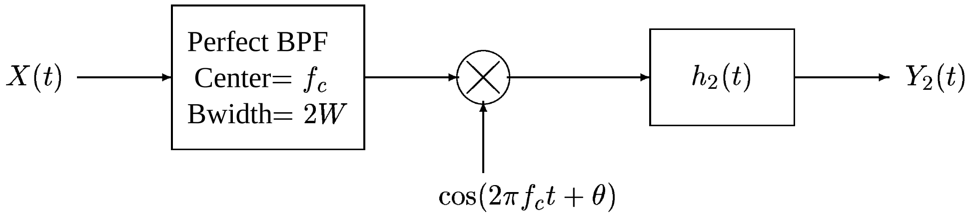
where $h_1(t)$ is a perfect lowpass filter of bandwidth W; that is, its frequency response is given by:

$$H_1(f) = \begin{cases} 1 & |f| \le W \\ 0 & \text{otherwise} \end{cases}$$

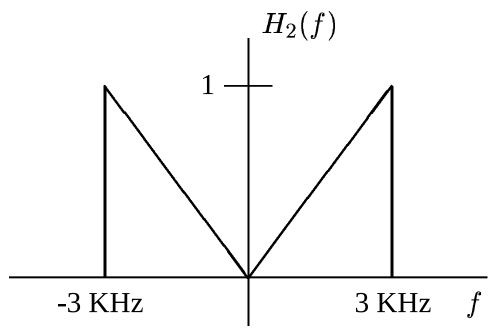
Find the signal-to-noise ratio (SNR) of $Y_1(t)$.

Problem #5 Continued. . .

[10] (d) Suppose instead that we process $X(t)$ with the following receiver:

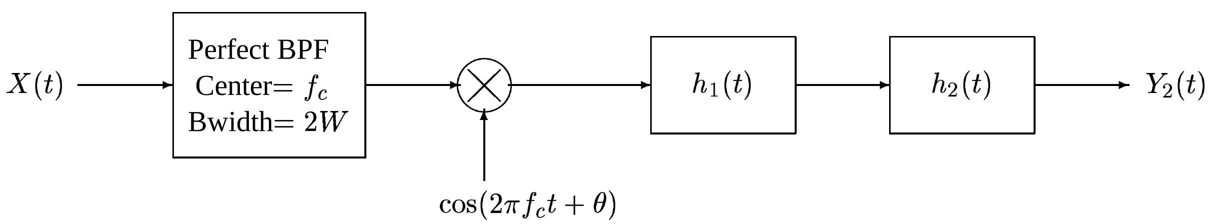


where the filter $h_2(t)$ is characterized by the frequency response:



Find the signal-to-noise ratio (SNR) of $Y_2(t)$. (Assume that any component in $Y_2(t)$ due to $M(t)$ is "signal", and any component in $Y_2(t)$ due to only $N(t)$ is "noise".)

[BIG HINT for 5(d): If you can justify why this is so, you can instead analyze the following equivalent system (which you may find easier).]



[5] (e) How much better (or worse) is the receiver in part (d) than that in part (c)?