ECE 566 - Communication Systems I
Fall 2000 Final Exam
Wednesday, December 20, 1:30-3:30 p.m., ELAB 304

Overview

- The exam consists of six problems for 110 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **three page-sides** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because . . . " so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an "F" for the course.

<u>Some potentially useful information</u>

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a \rvert} X\left(\frac{f}{a}\right) e^{j2\pi \frac{b}{a} f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
$x(t) = \begin{cases} 1 - \lvert t \rvert & \lvert t \rvert \leq 1 \\ 0 & \lvert t \rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

<u>Parseval's Relation:</u> If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. Suppose that we are attempting to design a circuit which upconverts the in-phase component $x_I(t)$ and quadrature component $x_Q(t)$ to yield the band-pass signal:

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

To do this, we need to generate $c(t) = \cos(2\pi f_c t)$ and $s(t) = \sin(2\pi f_c t)$, which can be done by employing an oscillator to generate $c(t)$ and then generating $s(t)$ as $s(t) = c(t - \tau)$.

[5] (a) Find the desired value for τ in terms of f_c.

Suppose that we are unable to get the exact value of τ found in part (a); instead, we get the delay $\tau + \tau_\epsilon$, where τ_ϵ is some small error, and thus:

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\cos(2\pi f_c(t - \tau - \tau_\epsilon)) \\ &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c(t - \tau_\epsilon)) \\ &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t - \theta) \end{aligned}$$

where $\theta = 2\pi f_c \tau_\epsilon$. One way to detect this small phase error θ is to input the signals $x_I(t) = \cos(2\pi f_m t)$ and $x_Q(t) = \sin(2\pi f_m t)$ into the circuit.

[15] (b) For a fixed θ, employ frequency domain techniques to figure out how θ can be determined from $|X(f_c + f_m)|$ and $|X(f_c - f_m)|$, where $X(f)$ is the Fourier transform of the circuit output $x(t)$ when the inputs $x_I(t) = \cos(2\pi f_m t)$ and $x_Q(t) = \sin(2\pi f_m t)$ are applied. You should assume $f_m \ll f_c$. *Hint: You might find the complex baseband representation useful here.*

[5] (c) Suppose $\theta = 0$. The output $x(t)$ when $x_I(t) = \cos(2\pi f_m t)$ and $x_Q(t) = \sin(2\pi f_m t)$ is equivalently what type of modulation by the message $\cos(2\pi f_m t)$ of the carrier $\cos(2\pi f_c t)$? (I will give you bonus points if you give a nice explanation of why this is true.)

2. The message signal $M(t)$, which is a zero-mean wide-sense stationary random process with autocorrelation function $R_M(\tau) = 2\ \mathrm{sinc}(5000\tau)$, is transmitted using DSB-SC; the transmitted waveform is given by:

$$X(t) = M(t)\cos(2\pi 10^6 t + \theta)$$

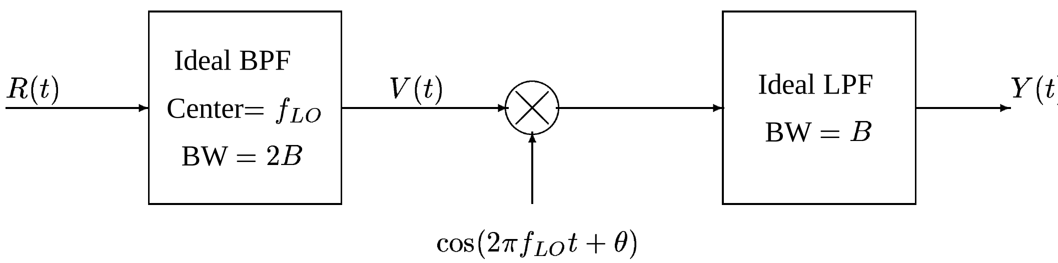
and the received signal is given by

$$R(t) = X(t) + W(t)$$

where $W(t)$ is a zero-mean wide-sense stationary random process with power spectral density $S_W(f) = \frac{1}{10^6}\Lambda(\frac{f}{2\cdot 10^6})$, where:

$$\Lambda(x) = \begin{cases} 1-x, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases} .$$

Assume that the **coherent** receiver given below is employed.



[5] (a) Provide the parameters f_{LO} and B such that the noise power at the output is minimized, but none of the signal is filtered out.

[10] (b) Find the signal-to-noise ratio at the output of the receiver using your specifications from (a).