ECE 566 - Communication Systems I, Fall 1998 Final Exam

Overview

- The exam consists of six problems for 110 points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **three page-sides** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "this must be wrong because . . . " so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

Some potentially useful information

$$\begin{aligned}\cos(\theta) &= \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \\ \sin(\theta) &= \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)\end{aligned}$$

$$\begin{aligned}\sin(a \pm b) &= \sin(a)\cos(b) \pm \cos(a)\sin(b) \\ \cos(a \pm b) &= \cos(a)\cos(b) \mp \sin(a)\sin(b)\end{aligned}$$

$$\begin{aligned}\cos(a)\cos(b) &= \tfrac{1}{2}\left[\cos(a-b) + \cos(a+b)\right] \\ \sin(a)\sin(b) &= \tfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \\ \sin(a)\cos(b) &= \tfrac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]\end{aligned}$$

Time Function	**Fourier Transform**
$x(at+b)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)e^{j2\pi\frac{b}{a}f}$
$p(t) = \begin{cases} 1 & \lvert t \rvert \leq 1/2 \\ 0 & \lvert t \rvert > 1/2 \end{cases}$	$P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$
$x(t) = \begin{cases} 1-\lvert t\rvert & \lvert t\rvert \leq 1 \\ 0 & \lvert t\rvert > 1 \end{cases}$	$X(f) = \text{sinc}^2(f)$

Parseval's Relation: If $X(f)$ is the Fourier Transform of $x(t)$,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

1. Remember to justify all answers. In all cases, “it is neither” or “it could be either” are possible answers.

   [5] (a) Is the signal $x(t)$ in Figure 1 the output of an analog communication transmitter or a digital communication transmitter?

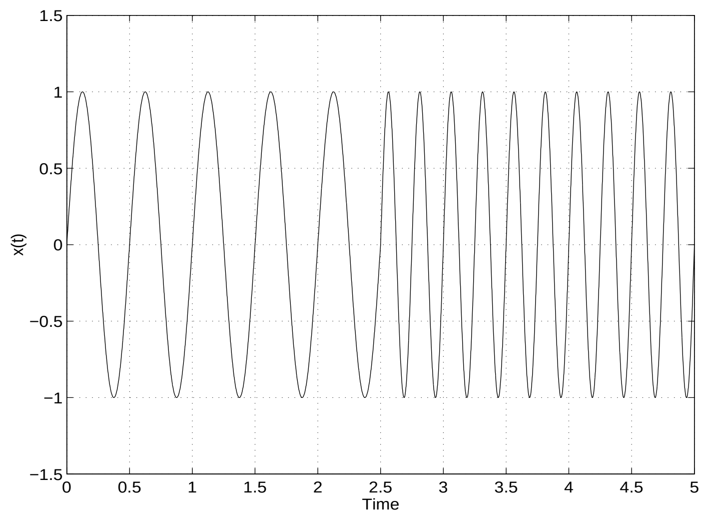


Figure 1: Signal $x(t)$ for Problem 1(a)

[5] (b) Is the analog communication signal $x(t)$ in Figure 2 the output of an amplitude modulation transmitter or an angle modulation transmitter? Further:

- If it is an amplitude modulation signal, tell whether it is DSB-SC, AMTC, or conventional AM.
- If it is an angle modulation signal, tell whether it is FM or PM.

[5 BONUS] (c) Estimate **roughly** the bandwidth of the signal in part (b). (*Hint: The message is a sinusoid.*)

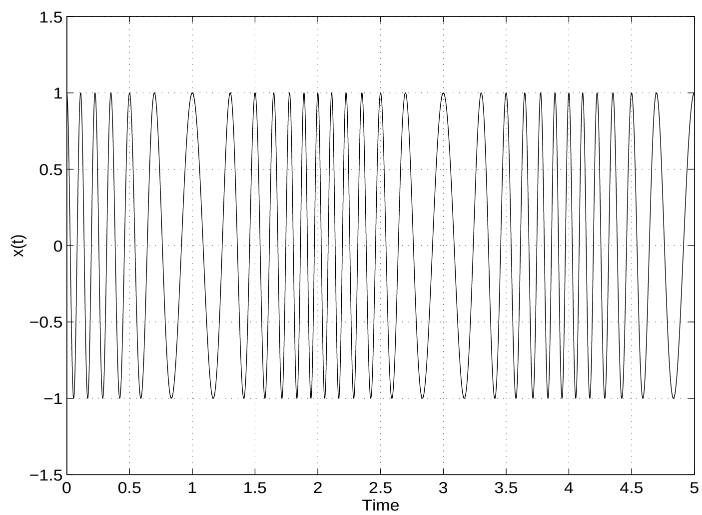


Figure 2: Signal $x(t)$ for Problem 1(b),1(c)

2. Suppose that we have a bandpass signal $x(t)$ of bandwidth 200 Hz around the carrier frequency 500 Hz. We desire to filter this signal with a bandpass filter with frequency response:

$$H(f) = \begin{cases} \frac{|f|-400}{200} & 400 \leq |f| \leq 600 \\ 0 & \text{otherwise} \end{cases}$$

[8] (a) Sketch $H_I(f)$ and $H_Q(f)$, the Fourier transforms of the in-phase and quadrature components of $h(t)$, respectively.

[7] (b) Draw a block diagram of a system with input $x(t)$ and output $h(t) * x(t)$ that uses only summers, multipliers, oscillators, and *lowpass* filters (i.e. filters that are lowpass with bandwidth W). Be sure to give the frequency response of *all* filters. Remember that all signals and filter impulse responses must be real, of course.

3. Some DSB-SC problems:

[10] (a) The deterministic message signal $m(t) = \text{sinc}(5000t)$ is modulated on a DSB-SC carrier to form the transmitted signal:

$$x(t) = m(t)\cos(2\pi 10^9 t)$$

Find the transmitted signal's Fourier transform $X(f)$, sketch $X(f)$, and find the energy in $x(t)$.

[10] (b) A deterministic message $m(t)$ that is lowpass with bandwidth W is modulated on a carrier to form the DSB-SC signal:

$$x(t) = m(t)\cos(2\pi f_c t)$$

where $f_c \gg W$.

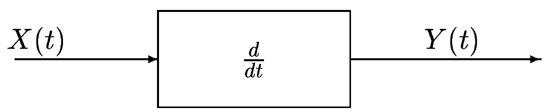
Suppose that we want to recover $m(t)$ from $x(t)$ using only a sampler (it can be ideal and you get to pick the sampling frequency) and a single LTI filter. Show how to do this - be sure to justify your answer.

4. Signal-to-noise ratio calculations:

[10] (a)Let $X(t) = m(t) + N(t)$, where $m(t) = A_c \cos(2\pi 100 t)$ is the signal component and $N(t)$ is a bandlimited (zero-mean) white noise process with power spectral density

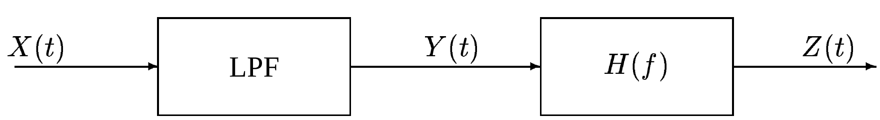
$$S_N(f) = \begin{cases} \frac{N_0}{2} & |f| \leq 1000 \\ 0 & \text{otherwise} \end{cases}$$

$X(t)$ is input to an ideal differentiator as shown below.



- Find the **signal-to-noise ratio** at the input to the ideal differentiator (i.e. the signal-to-noise ratio of $X(t)$).
- Find the **signal-to-noise ratio** at the output of the ideal differentiator (i.e. the signal-to-noise ratio of $Y(t)$).

[10] (b) Let $X(t) = M(t) + W(t)$, where $M(t)$ is a zero-mean random signal with autocorrelation function $R_X(\tau) = \text{sinc}(400\tau)$ and $W(t)$ is a (zero-mean) white noise process with power spectral density $S_W(f) = \frac{N_0}{2}$. $X(t)$ is run through an ideal lowpass filter with bandwidth 1000 Hz and then through an LTI filter with frequency response $H(f) = e^{-\frac{f^2}{8}}$ as shown below. (As usual, assume $M(t)$ and $W(t)$ are independent).



- Find the **signal-to-noise ratio** at the output of the lowpass filter (i.e. the signal-to-noise ratio of $Y(t)$).
- Find the **noise power** at the output of the LTI filter (i.e. the noise power in $Z(t)$).