

ECE 603 - Probability and Random Processes, Fall 2006

Homework #8

Due 11/22/06

1. A fair coin is tossed. If “heads” appears, $X[n] = 1$ for all n . If “tails” appears, $X[n] = (-1)^n$ for all n .
 - (a) Find the probability density function $f_{X[n]}(x)$ (Note that $X[n]$ is a discrete random variable.)
 - (b) Find the mean function $m_X[n]$.
 - (c) Find the autocorrelation function $R_X[m, n]$.

2. Let $Z(t) = Xt + Y$, where $X \sim N(0, 2)$ and $Y \sim N(1, 1)$, and X and Y are independent.
 - (a) Find the probability density function $f_{Z(t)}(x)$.
 - (b) Find the mean function $m_Z(t)$.
 - (c) Find the autocorrelation function $Z_Z(t_1, t_2)$.

3. Define

$$g(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{else} \end{cases} .$$

Define the random process $X(t) = g(t - T)$, where T is uniformly distributed between 0 and 1.

- (a) Find the probability density function $f_{X(t)}(x)$. (Note that $X(t)$ is a discrete random variable.)
- (b) Find the mean function $m_X(t)$.
- (c) Find the autocorrelation function $R_X(t_1, t_2)$.

4. In a simple quadrature amplitude modulation (QAM) communication system, $X(t)$ is defined as:

$$X(t) = A \cos(2\pi f_c t) + B \sin(2\pi f_c t)$$

where f_c is the “carrier” frequency, and A and B are **independent** random variables with mean zero and variance σ^2 .

- (a) Find the mean function of $X(t)$.
- (b) Find the autocorrelation function of $X(t)$.
- (c) Is $X(t)$ wide-sense stationary?
- (d) Is $X(t)$ strict-sense stationary? (*Hint: To show that it is not, try showing that $E[X^3(t)]$ depends on t .*)

5. A discrete-time random process is defined as:

$$X[n] = \frac{1}{\sqrt{2}}X[n-1] + U[n], \quad n = 1, 2, 3, \dots$$

where $X[0] \sim N(0, 1)$ and the random process $U[n]$ consists of independent and identically distributed (IID) random variables, with $U[n] \sim N(0, \frac{1}{2})$ for all n .

- (a) Find the mean function of $X[n]$.
- (b) Find the variance of $X[n]$ for all n .
- (c) Find the autocorrelation function of $X[n]$.
- (d) Is $X[n]$ wide-sense stationary?