ECE 603 - Probability and Random Processes, Fall 2006

Homework #7

Due 11/10/06

1. Use the Central Limit Theorem to prove the Weak Law of Large Numbers.

2. A fair die is tossed 100 times:

   (a) Using Cheybshev's inequality, find a bound on the probability that the total number of spots (sum of faces that show) is between 316 and 384.

   (b) Using the Central Limit Theorem, estimate the probability that the total number of spots if between 316 and 384. Compare your answer to part (a).

3. (a) Let $X_1, X_2, X_3, \ldots$ be independent Gaussian random variables, each with density function $f_X(x)$ with unknown mean $E[X] = \mu$ and variance 1.

   - Suppose that we observe 100 variables in this sequence and estimate the mean μ as:

   $$\hat{\mu} = \frac{1}{100} \sum_{i=1}^{100} X_i = 4.5$$

     Give an interval $[a, b]$ such that $P(\mu \in [a, b]) = 0.99$.
   - Find the minimum number of samples N that we must take such that $|\mu - \frac{1}{N} \sum_{i=1}^{N} X_i| < 0.05$ with: (i) 95% confidence, (ii) 99% confidence

   (b) You are attempting to estimate the probability of an event A. Find the number of samples you would have to take to get within ± 0.01 accuracy with 95 % confidence if: (i) $P(A) = 0.5$, (ii) $P(A) = 0.01$.