ECE 603 - Probability and Random Processes, Fall 2006

Homework #3

Due 10/06/06

1. In a box there are 20 parts, 3 of which are defective. You draw parts from the box one after another without replacement and test them to find the defective ones. Find the probability of the following:

   (a) You have drawn and tested 10 parts and you have not observed any defective parts.

   (b) Within the 10 parts drawn/tested, you have observed 2 defective parts.

   (c) On the 10th draw/test, you observe the third defective part.

2. In each trial of an experiment, an error occurs with probability 0.15. An error is a *major* error with probability 0.2 and a *minor* error with probability 0.8. The experiment is repeated 12 times, and trials are independent of one another.

   (a) What is the probability that an error occurs in exactly 4 of the trials?

   (b) What is the probability that exactly 3 minor errors occur?

   (c) What is the probability that exactly 2 minor and 2 major errors occur?

   (d) What is the number of times the experiment has to be repeated to have 6 or more error-free trials with probability greater than or equal to 0.9?

3. Two real numbers b and c are chosen completely at random and independently of each other. What is the probability that the quadratic equation $t^2 + bt + c = 0$ has real roots? *(Hint: First solve this problem for b and c each drawn from $(-n, n)$ and then take the limit.)*

4. (a) Suppose a stick of length L is broken into two pieces at a random point along its length, all points equally likely. Let the random variable X represent the length of the shorter one of the two pieces. Find the cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$ for X.

   (b) Repeat part (a), if the likelihood of a breaking point is proportional to the square of the distance between the point and the closer of the two endpoints.

   (c) Repeat part (a), but now find the cumulative distribution function $F_Y(y)$ and probability density function $f_Y(y)$ for the random variable Y if all break points are equally likely (as in (a)), but with Y defined as the difference of the lengths of the long and the short pieces.

5. A service facility charges a \$20 fixed fee plus \$25 per hour of service up to 6 hours, and no additional fee is charged for service exceeding 6 hours. Suppose that the service time τ is equally likely to be any time in $[0, 10]$ hours (Note that τ is a continuous random variable). Let X represent the cost of service in the facility.

   (a) Find *and sketch* the cumulative distribution function for X.

(b) Find *and sketch* the probability density function for X.

(c) What is the probability that you end up paying less than \$60 for service? Answer this part two different ways:

- Using your answer to part (b).
- Finding the time (call it τ_0) at which the service would cost *exactly* \$60 and then finding the probability that $\tau \leq \tau_0$.