ECE 563 - Introduction to Comm/SP, Fall 2006
Homework #6

Due: 12/14/07

1. Suppose we have a signal that is bandlimited to 5 kHz. We sample the signal at a rate of 12000 samples/second. We want to examine the signal's frequency components using a DFT.

   (a) If the DFT length must be a power of 2 and the analog frequency spacing between adjacent DFT samples can be no more than 5 Hz, what is the minimum DFT length N that can be used?

   (b) For the N that you found in part (a): what DFT sample number k is closest to representing an analog frequency of 4600 Hz?

2. A random variable X has pdf:

$$f_X(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{x^2}{8}}$$

   (a) Find $E[X]$ and $E[X^2]$.

   (b) Find $P(X > 4)$.

   (c) Let $Y = 4X - 2$. Find $P(Y < -3)$.

3. Let $X(t)$ be a wide-sense stationary Gaussian random process with mean zero and autocorrelation $R_X(\tau) = e^{-\frac{|\tau|}{2}}$. Let $N(t)$ be a white Gaussian noise process with power spectral density $\frac{N_0}{2}$.

   (a) Find P_x, the power in $X(t)$.

   (b) Find $P(X(3) > 1)$.

   (c) Find the power spectral density $S_X(f)$ of $X(t)$.

   (d) Find a filter (give $h(t)$ or $H(f)$) that has input $N(t)$ and output with power spectral denisty $S_X(f)$.

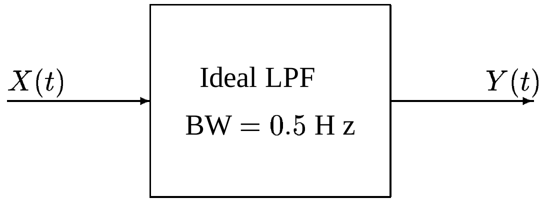
   (e) Let $Z = X(0) + X(1) + X(2)$. Find $f_Z(z)$, the pdf of Z.

   (f) Find $P(X(0) + X(2) > 3)$.

4. *(Note: This has been an exam question in the past.)*

Suppose that $X(t)$ is a zero-mean Gaussian random process with autocorrelation function $R_X(\tau) = \text{sinc}^2(\tau)$.

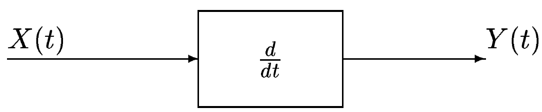
(a) Suppose that I run $X(t)$ through an ideal lowpass filter with unity gain and bandwidth 0.5 Hz (**Be sure you take into account the bandwidth!**) as shown below:



Find:

- $S_Y(f)$, the power spectral density of $Y(t)$.
- P_Y, the power in $Y(t)$.

(b) Suppose that I run $X(t)$ through an ideal differentiator as shown below:



Find:

- $S_Y(f)$, the power spectral density of $Y(t)$.
- P_Y, the power in $Y(t)$.

5. *(Note: This has been an exam question in the past.)*

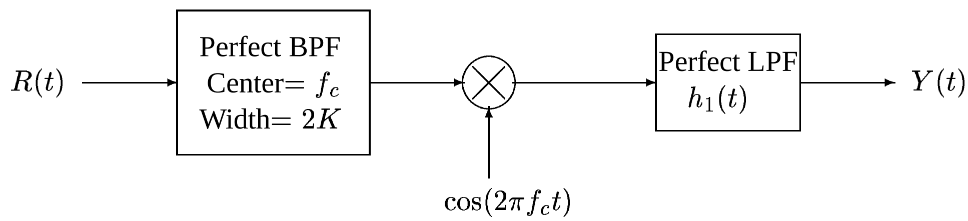
Let the message signal $m(t) = \text{sinc}(5t)$ amplitude modulate the carrier $15\cos(2\pi f_c t)$ to form the transmitted signal:

$$x(t) = 15\text{sinc}(5t)\cos(2\pi f_c t)$$

(a) Find (in any order):

- The power in $x(t)$.
- The energy in $x(t)$.
- $X(f)$, the Fourier Transform of $x(t)$.
- A sketch of $X(f)$, the Fourier Transform of $x(t)$.
- The bandwidth of $x(t)$.

The received signal $R(t) = x(t) + W(t)$, where $W(t)$ is a zero-mean Gaussian white noise process with power spectral density $S_W(f) = \frac{N_0}{2}$, is input to the following receiver:



where $h_1(t)$ has frequency response given by:

$$H_1(f) = \begin{cases} 1, & |f| \leq K \\ 0, & \text{otherwise} \end{cases}$$

and K is the bandwidth of $m(t)$.

(b) Characterize the signal portion of $Y(t)$ in the frequency domain (use the frequency-domain tool appropriate for this type of signal). Find the energy in the signal portion of $Y(t)$.

(c) Characterize the noise portion of $Y(t)$ in the frequency domain (use the frequency-domain tool appropriate for this type of signal). Find the power in the noise portion of $Y(t)$.

6. Suppose that we have a message signal $M(t)$, which is a lowpass process of bandwidth W, that we transmit with DSB-SC:

$$X(t) = A_c M(t) \cos(2\pi f_c t)$$

The received signal is given by $R(t) = X(t) + W(t)$, where $W(t)$ is a zero-mean white noise process with power spectral density $S_W(f) = \frac{N_0}{2}$. Suppose that we use the standard receiver for DSB-SC in noise as given in class, but that the local oscillator at the receiver has a phase error; that is, the local oscillator is given by $\cos(2\pi f_c t + \phi)$.

(a) Assuming ϕ is fixed, derive an expression for the receiver output $Y(t)$ as a function of $M(t)$, ϕ, $N_I(t)$, and $N_Q(t)$ (where $N_I(t)$ and $N_Q(t)$ are the in-phase and quadrature components, respectively, of the narrowband noise at the output of the BPF).

(b) Assuming that ϕ and $M(t)$ are fixed, find the expected value of $Y(t)$ as a function of ϕ and $M(t)$.

(c) Assuming that ϕ is fixed, find the output SNR as a function of ϕ and P_m, the power in the message.

(d) Assuming that ϕ is uniformly distributed between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$, find the average output SNR.