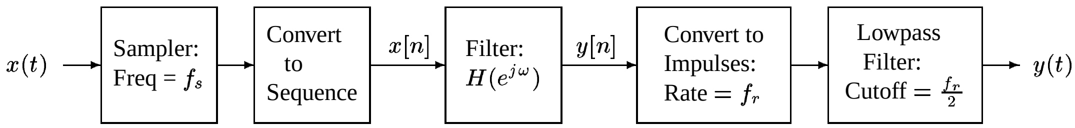
ECE 563 - Introduction to Comm/SP, Fall 2007
Homework #5

Due: 11/12/07

1. Consider the following system to process continuous-time signals with discrete-time processing.



(a) Suppose that $x(t) = \cos(2\pi 2500t)$, $f_s = f_r = 10000$ Hz, and the discrete-time filter frequency response, which you recall is periodic with period 2π, is given in $[-\pi, \pi]$ as:

$$H(e^{j\omega}) = \frac{\omega^2}{\pi^2}, \qquad |\omega| < \pi$$

Find the output $y(t)$.

(b) Suppose that $x(t)$ has Fourier Transform $X(f) = p(f/10000)$, $f_s = f_r = 10000$ Hz, and the discrete-time filter frequency response, which you recall is periodic with period 2π, is given in $[-\pi, \pi]$ as:

$$H(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{2} < |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$$

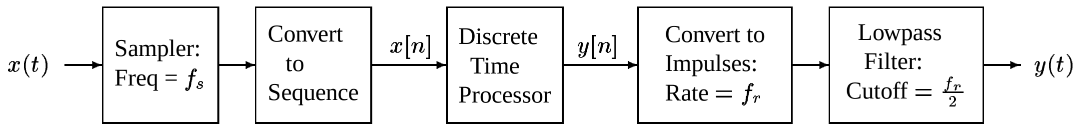
Find the Fourier transform $Y(f)$ of the output $y(t)$, and then use it to find $y(t)$.

(c) Suppose that $x(t) = \cos(2\pi 2500t)$, $f_s = f_r = 10000$ Hz, and the discrete-time filter is defined by the difference equation:

$$y[n] = 0.5y[n-1] + x[n]$$

Find the output $y(t)$ [*Note: You may get some weird trigonometric thing (e.g.* $\cos^{-1}(\frac{3}{7})$*) that you cannot evaluate without your calculator. Just leave it in its trigonometric form (e.g.* $\cos^{-1}(\frac{3}{7})$*) in your answer.*]

2. Consider the following system to process continuous-time signals with discrete-time processing.



Your discrete-time processing toolkit on your computer has the following four blocks for possible use (when a frequency response is given, it is valid for $\omega \in [-\pi, \pi]$, and it repeats outside of that range, of course):

Block 1: $H_1(e^{j\omega}) = e^{-j\omega n_0}$, where you can choose any n_0 that you wish

Block 2: $H_2(e^{j\omega}) = j\omega$

Block 3: $H_3(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{2} \\ 0, & \text{else} \end{cases}$

Block 4: The fourth block provides output $y[n] = (-1)^n x[n]$ for input $x[n]$.

Using any combinations of these blocks (including multiple versions of any, if needed) and specifying the sampling frequencies f_s and f_r, provide a separate system to implement each of the following desired continuous-time operations with the block diagram above.

(a) The lowpass filter $H(f) = p(f/2000)$, to be applied on input signals $x(t)$ of bandwidth up to 1.5 KHz to yield the signal $y(t)$.

(b) A system that delays the input $x(t)$, of bandwidth less than 10 KHz, by 2×10^{-3} seconds to yield the signal $y(t)$.

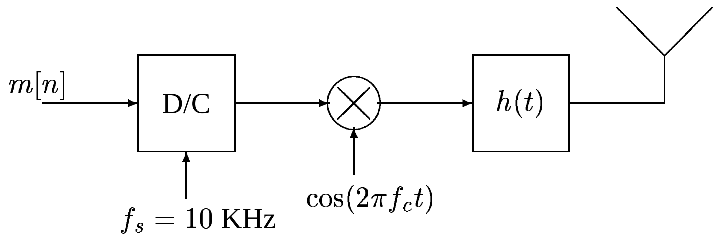
(c) A system that yields $y(t) = \frac{d}{dt}x(t)$ for inputs $x(t)$ of bandwidth less than 10 KHz.

(d) A system that implements the bandpass filter:

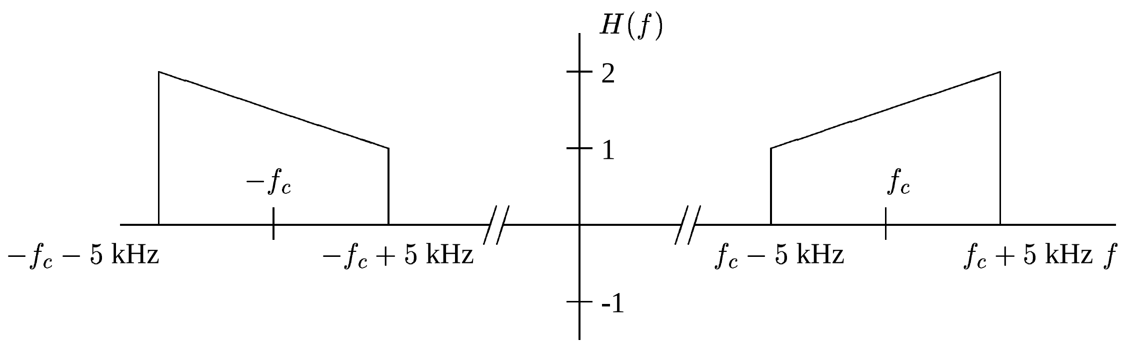
$$H(f) = \begin{cases} 1, & 5000 < |f| < 10000 \\ 0, & \text{else} \end{cases}$$

on inputs signals $x(t)$ of bandwidth less than 10 KHz to yield the signal $y(t)$.

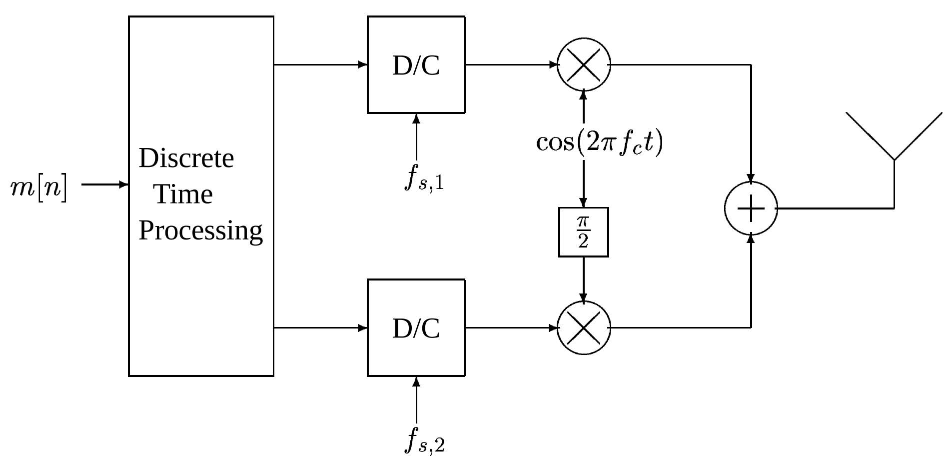
3. A voice signal $m(t)$ of bandwidth 5 KHz is sampled at 10 KHz to yield the discrete-time signal $m[n]$. We desire to transmit this discrete-time voice signal using double-sideband suppressed carrier with some passband filtering by a filter $h(t)$. One possible method of doing such is:



where $h(t)$ has Fourier Transform:

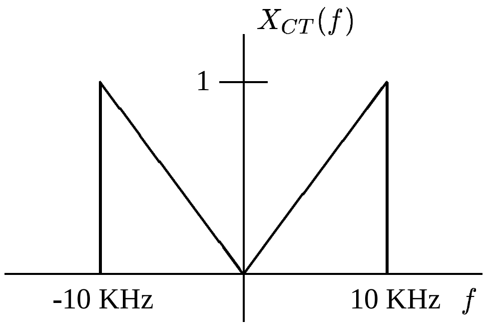


However, we have a tough time building bandpass filters and thus would like to do the filtering at (complex) baseband. Furthermore, since $m[n]$ is already discrete-time, we might as well do the filtering in discrete-time, which allows us to use the following circuit:



Find $f_{s,1}$, $f_{s,2}$, and the discrete-time processing such that this circuit yields the same output as the circuit in the first diagram above.

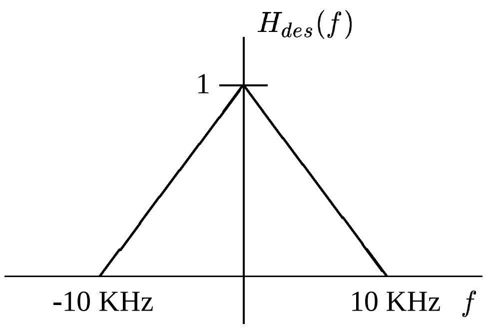
4. The company you work for builds a radar to collect data on a signal $x(t)$ with Fourier transform:



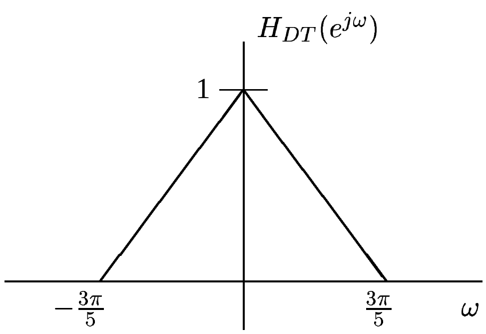
The radar samples (using an ideal sampler) the signal $x(t)$ at a rate of 25 KHz to produce the discrete-time sequence $x[n]$, which is recorded. **Note that the waveform $x(t)$ is no longer available, since the radar only recorded $x[n]$.**

(a) Find $X_{DT}(e^{j\omega})$, the Fourier Transform of the discrete-time sequence $x[n]$.

Your boss comes to you and tells you that she wishes that you had lowpass filtered $x(t)$ with the filter $h_{des}(t)$ before you sampled it. The filter $h_{des}(t)$ has frequency response:



You search the WWW and are able to find coefficients for a very good discrete-time filter $h_{DT}[n]$ with a frequency response, which you recall is periodic with period 2π, given in $[-\pi, \pi]$ as:



(b) Using discrete-time processing with only upsamplers, downsamplers, perfect discrete-time low-pass filters and the discrete-time filter $h_{DT}[n]$, give a block diagram for processing $x[n]$ to obtain the sequence $y[n]$ that your boss desires: that which would have been the result of sampling $h_{des}(t) * x(t)$ at 25 KHz.

5. In this problem, you are going to consider how to do multiplication of two continuous-time signals using discrete-time processing.

(a) Find the Fourier transform $X(f)$ of:

$$x(t) = 5000\text{sinc}(10000t)$$

and the Fourier transform $Y(f)$ of

$$y(t) = 5000\text{sinc}(40000t)$$

(b) Find the bandwidth (you do not have to find the full Fourier transform unless you would like) of the signal $z(t) = x(t)y(t)$.

(c) Your buddy samples the signal $x(t)$ at 15 KHz to obtain the sequence $x[n]$ and the signal $y(t)$ at 45 KHz to obtain the signal $y[n]$. Give the discrete-time processing of $x[n]$ and $y[n]$, and the sampling frequency f_r, for the D/C conversion such that the circuit below outputs $z(t) = x(t)y(t)$. [*Notes: (1) Be sure that your circuit would work for any $x(t)$ and $y(t)$ of the same bandwidth as the signals given. (2) You have all discrete-time blocks for your use, including discrete-time filters of any type, upsamplers, downsamplers, mathematical operations of any type, etc.*]