ECE 563 - Introduction to Comm/SP, Fall 2007
Homework #3

Due: 10/13/07

1. Suppose we run a real bandpass signal $x(t)$, whose Fourier transform is nonzero only for $f_c - W \leq |f| \leq f_c + W$ (assume $W \ll f_c$), through a Hilbert Transformer $h(t)$, whose frequency response is given by:

$$H(f) = \begin{cases} -j, & f \geq 0 \\ j, & f < 0 \end{cases}$$

to arrive at $\hat{x}(t) = x(t) * h(t)$, which is defined as the *Hilbert Transform* of $x(t)$.

(a) Show that the Hilbert Transform of $x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$ is given by $\hat{x}(t) = x_I(t)\sin(2\pi f_c t) + x_Q(t)\cos(2\pi f_c t)$. (*Hint: Do this in the frequency domain.*)

(b) Define the signal $z(t) = x(t) + j\hat{x}(t)$. Using part (a), show that the complex envelope $x_z(t)$, as defined in class, is given by: $x_z(t) = z(t)e^{-j2\pi f_c t}$. (*Hint: Do this in the time domain.*)

2. The bandpass signal $x(t) = \text{sinc}(t)\cos(2\pi f_c t)$ is passed through a bandpass filter with impulse response $h(t) = \text{sinc}^2(t)\sin(2\pi f_c t)$ to yield the output $y(t)$.

(a) Find the complex envelopes of $x(t)$ and $h(t)$ (i.e. find $x_z(t)$ and $h_z(t)$, respectively).

(b) Find and sketch the Fourier transforms of $x_z(t)$ and $h_z(t)$.

(c) Find $y_z(t)$, the complex envelope of $y(t)$, and use it to find $y(t)$.

3. Suppose that I have a signal $x(t)$ with Fourier transform

$$X(f) = \frac{1}{400}p\left(\frac{f - f_c}{200}\right) + \frac{1}{400}p\left(\frac{f + fc}{200}\right)$$

where $f_c \gg 1000$.

(a) Find real lowpass signals $x_I(t)$ and $x_Q(t)$ such that

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

(b) Now suppose that I filter the signal $x(t)$ with a filter with impulse response $h(t)$, where the Fourier transform of $h(t)$ is given by:

$$H(f) = \begin{cases} 1, & |f| \leq f_c \\ 0, & |f| > f_c \end{cases}.$$

Let the output of the filter be denoted $y(t) = h(t) * x(t)$. Find real lowpass signals $y_I(t)$ and $y_Q(t)$ such that:

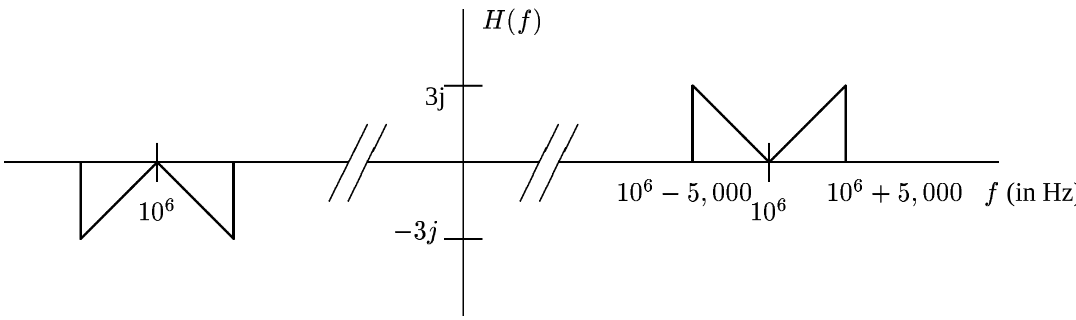
$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

4. In your first job after graduation from UMass, you are assigned to design a transmitter that outputs $y(t) = h(t) * x(t)$, where

$$x(t) = x_I(t)\cos(2\pi 10^6 t) - x_Q(t)\sin(2\pi 10^6 t)$$

is a bandpass signal with lowpass signals $x_I(t)$ and $x_Q(t)$ (each of bandwidth 5 KHz) as its in-phase and quadrature components, respectively, and $h(t)$ is a real bandpass filter. The inputs to your transmitter are $x_I(t)$ and $x_Q(t)$, and the output is $y(t)$.

(a) Suppose that the bandpass filter response is specified by $H(f)$, the Fourier transform of $h(t)$:



Sketch the Fourier transforms $H_I(f)$ and $H_Q(f)$ of the in-phase part $h_I(t)$ and quadrature part $h_Q(t)$, respectively, of the filter $h(t)$. *Hint:* Recall from class that

$$\begin{aligned} H_I(f) &= \frac{H_Z(f) + H_Z^*(-f)}{2} \\ H_Q(f) &= \frac{H_Z(f) - H_Z^*(-f)}{2j}, \end{aligned}$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

(b) Draw a circuit that takes as input $x_I(t)$ and $x_Q(t)$ and outputs $y(t)$, while employing only summers, multipliers, oscillators, and *lowpass* filters. (*Note:* A "lowpass filter" is defined for this problem as one whose frequency response is non-zero only for $|f| \leq 5KHz$.)

(c) Find the output $y(t)$ of your transmitter when $x(t) = 6\cos(2\pi 1,002,500t + \frac{\pi}{2})$.

(d) Suppose that $X(f) = H(f)$. Determine whether $x(t)$, the inverse Fourier transform of $X(f)$, could be the output of a DSB-SC system; that is, of the form

$$x(t) = A_c m(t)\cos(2\pi f_c t + \theta)$$