

ECE 563 - Introduction to Comm/SP, Fall 2007

Homework #2

Due: 10/03/07

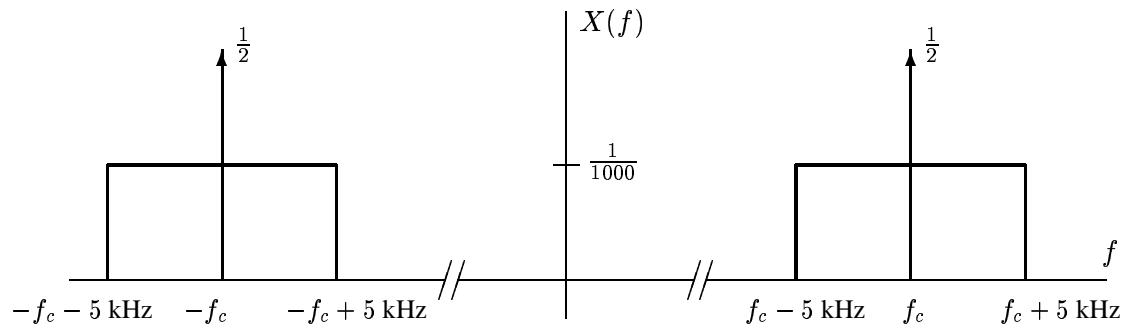
1. Let $x_1(t)$ be the modulated signal when $m_1(t)$ is the message signal. Let $x_2(t)$ be the modulated signal when $m_2(t)$ is the message signal. A type of modulation is said to be *linear* if $\alpha_1 x_1(t) + \alpha_2 x_2(t)$ is the modulated signal for the message $\alpha_1 m_1(t) + \alpha_2 m_2(t)$ for all $\alpha_1, \alpha_2, m_1(t)$, and $m_2(t)$. Determine whether the following modulations are linear:

(a) $x(t) = A_c m(t) \cos(2\pi f_c t)$

(b) $x(t) = A_c [1 + \alpha m(t)] \cos(2\pi f_c t)$

(c) $x(t) = A_c \cos(2\pi f_c t + k_p m(t))$

2. An amplitude modulation (AM) transmitter outputs a signal $x(t)$ with Fourier transform $X(f)$ given by:



The carrier frequency is $f_c = 1$ MHz. You also know that the message spectrum $M(f)$ contains no $\delta(\cdot)$ functions.

(a) Find $x(t)$ and then *roughly* sketch it. [In your drawing, focus on conveying the key points rather than artistic excellence.]

(b) Find the bandwidth of the signal $x(t)$.

(c) Suppose I remove the unmodulated carrier component from $x(t)$, leaving just a term of the form $A_2 m(t) \cos(2\pi f_c t)$. Find the energy in the resulting signal.

3. We use the message signal $m(t) = 6 \operatorname{sinc}^2(3t)$ to amplitude modulate a carrier of frequency $f_c = 8$ Hz. The modulation index is 0.8, and the unmodulated carrier (i.e. the output signal if we had set $m(t) = 0$) would have power 8.
- Write an expression for the resulting AM signal $x(t)$.
 - Draw a *rough* sketch of $x(t)$.
 - Find E_m , the energy in the message signal $m(t)$.
 - Find P_x , the power in the resulting AM signal $x(t)$. What is the power efficiency of this AM system?
 - Find $X(f)$ and *roughly* sketch $|X(f)|$, where $X(f)$ is the Fourier transform of the signal $x(t)$. What is the bandwidth of the signal $x(t)$?

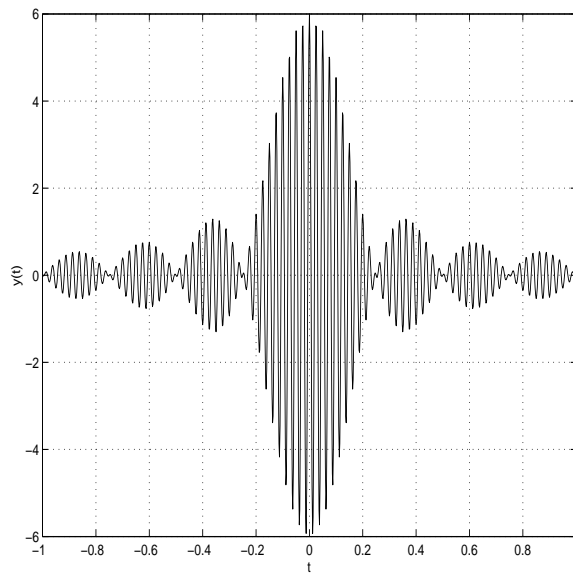


Figure 1: Signal $x(t)$ for Problem 4

4. Consider the AM signal $x(t)$ shown in Figure 1. You can assume that it continues infinitely in each direction with the same behavior shown in $1 \leq t \leq 2$.
- Write an expression for $x(t)$.
 - Find the Fourier transform $X(f)$.
 - Estimate the power in the signal $x(t)$.
 - Estimate the modulation index and bandwidth of the signal $x(t)$.

5. You desire to send the message signal

$$m(t) = 4 \cos(2\pi 5000t)$$

with an amplitude modulation (AM) system.

(a) Suppose that you choose to modulate your message onto a carrier signal $A_c \cos(2\pi 10^6 t)$ of power 8 (i.e. if the message were zero, the power of the transmitted signal would be 8), and that you want to employ a modulation index of 0.75. Write an expression for the transmitted signal $x(t)$, and roughly sketch $x(t)$.

(b) Provide the simplest possible receiver that can be used to recover $m(t)$. Be sure to provide all necessary parameters (e.g. filter bandwidths, oscillator frequencies, critical component values, etc.).

(c) Disaster strikes! After you have already built your transmitter from (a) and your receiver from (b), you find out that your system is supposed to operate instead with carrier frequency 2×10^6 Hz rather than 10^6 Hz. However, you do not have any more oscillators. Instead, all you have is a square-law device $y(t) = x^2(t)$ and money to buy a single linear time-invariant (LTI) filter (of your choice). Can you take your transmitter from (a), possibly adjust the choice of a , and *run the output of the transmitter from (a)* through these two additional components to make a transmitter that works at $f_c = 2 \times 10^6$ Hz with a conventional AM receiver?

(d) Suppose in part (c) that you were able to build a transmitter which perfectly accomplished its goal: doubling the carrier frequency to 2×10^6 Hz to form a signal that can be received with a conventional AM receiver with no additional distortion to your message. What changes are required for the receiver in (b) to receive this signal? Because you desire to save money (and you already have the receiver in (b)), make as few changes as possible.