ECE 563 - Introduction to Comm/SP, Fall 2006
Homework #1

Due: 9/29/07 (but Homework #2 is posted 9/27/07)

1. Euler's Identities are give by:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

Use Euler's identities to prove the following trigonometric identities:

(a) $\sin(2\theta) = 2\sin\theta\cos\theta$

(b) $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

(c) Use the result from (b) to show that:

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

(d) $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

(e) Use the result from (d) to show that:

$$\sin\theta\cos\phi = \frac{1}{2}\sin(\theta+\phi) + \frac{1}{2}\sin(\theta-\phi)$$

And, now that you have derived them, try to remember these important identities - that is the point of this simple problem.

2. The system defined by

$$y(t) = \frac{1}{\sqrt{T}}\int_{t-T}^{t} x(\tau)d\tau$$

is a normalized finite-time integrator. Is this a linear time-invariant system? If your answer is yes, find the system impulse response.

3. (a) Can an LTI system produce the output $y(t) = \text{sinc}^2(t)$ when the input is $x(t) = \text{sinc}(t)$? Be sure to justify your answer.

(b) Consider the convolution of a signal $a(t)$ with itself to yield $b(t) = a(t) * a(t)$. Does there exist a signal $a(t)$ such that $a(t) = b(t) = a(t) * a(t)$? *Note: Be sure to justify your answer.*

(c) Let the signal $y(t)$ be a real lowpass signal with Fourier transform $Y(f)$ that is non-zero for $|f| < W$ and zero otherwise. Suppose I run $y(t)$ through a square-law device to yield $z(t) = y^2(t)$. What is the maximum bandwidth of $z(t)$? *Note: A justification with a few words and a good picture is fine here.*

4. Moving stuff around in the frequency domain is generally done by multiplying by a sinusoid and then filtering. Suppose that I have the signal $x(t) = \cos(2\pi 30t)$. I wish to multiply this by a sinusoid, apply a single LTI filter, and end up with the result $x(t) = \cos(2\pi 20t)$. Give **two** different frequencies for the sinusoid that can accomplish this (along with the corresponding LTI filter in each case).

5. A message signal $m(t)$ of energy E_m and Fourier transform $M(f)$, which is non-zero only for $f \in [-W, W]$, is to be transmitted via a type of amplitude modulation. Let the transmitted signal be given by $x(t) = m(t)\cos(2\pi f_c t)$, where $f_c \gg W$. Assume no noise in the system, and, hence the received signal $r(t)$ is equal to the transmitted signal ($r(t) = x(t)$).

   At the receiver, I want to process the received signal $r(t)$ to get back $m(t)$.

   (a) Find the energy E_x in the transmitted signal $x(t)$.

   (b) Suppose I process the signal $r(t)$ by multiplying by $\cos(2\pi f_c t)$ and then running the result through a perfect lowpass filter of bandwidth W (i.e. the filter response is 1 for $|f| < W$ and 0 else). Using frequency domain arguments, show that the output of the lowpass filter is (a possibly scaled version of) the message signal $m(t)$. Find the energy in the output of the lowpass filter as a function of the message energy E_m.

   (c) Instead, suppose that I process the signal by multiplying by $\cos(2\pi f_c t + \theta)$, where θ is a fixed constant, (i.e. there is a constant phase error in the receiver), and then running the result through a perfect lowpass filter of bandwidth W (i.e. the filter response is 1 for $|f| < W$ and 0 else). Find the output of the lowpass filter using time domain arguments. Find the energy in the output of the lowpass filter as a function of θ and the message energy E_m.

   (d) On average, the loss in output energy is not much - less than you need to add at the transmitter to form conventional AM and fix the problem. Then why is this a problem for broadcast AM radio? *Hint: Think about the user experience.*