ECE 313 - Signals and Systems, Fall 2012

Practice Problems - Midterm Exam #2
Remember also to look at the Practice Problems from Exam #1!

1. (a) Suppose that a discrete-time sequence $x[n]$ is given by:

$$\begin{aligned} x[-2] &= 2 \\ x[-1] &= 0 \\ x[0] &= -2 \\ x[1] &= -3 \\ x[2] &= 4 \\ x[3] &= 1 \\ x[4] &= 1 \end{aligned}$$

and $x[n] = 0$, otherwise. Without evaluating the full transform (which is unnecessary), use the definition and properties of the DTFT to find:

- $X(0)$
- $X(\pi)$
- $\int_{-\pi}^{\pi} X(\Omega)d\Omega$
- $\int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega$
- $\int_{-\pi}^{\pi} \left|\frac{dX(\Omega)}{d\Omega}\right|^2 d\Omega$

(b) Suppose that you have a discrete-time linear time-invariant (LTI) system for which somebody has given you the step-response; that is, they have given you the system response $g[n]$ to the input:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

How would you find the impulse response $h[n]$ of the LTI system? *Hint: Do this in the time domain. Think about how to exploit the linearity and time-invariance to obtain such.*

2. We know that a linear time-invariant (LTI) system is completely characterized by its impulse response $h[n]$. Suppose that the impulse response of our softward module "FILTER" is given by:

$$h[n] = 0.5\delta[n] + \delta[n-1] + 0.5\delta[n-2]$$

where $\delta[n]$ is the standard discrete impulse function.

(a) Is this system causal? Is it stable?

(b) To test our code, we first give input $x_b[n] = \text{rect}(n/5)$ to the module "FILTER" and obtain the output $y_b[n]$. Find the output $y_b[n]$.

(c) Find the frequency response $H_{DT}(\Omega)$ of "FILTER" and plot its magnitude and phase.

(d) Suppose I put the signal $x[n] = \cos(\frac{2\pi}{3}n)$ into the filter. Find the output $y[n]$.

3. Use trigonometric identities, Euler's formula, and the shifting property of the discrete-time Fourier transform (DTFT) to find the sequence $x_1[n]$ whose Fourier transform is given by:

$$X_1(\Omega) = 1 + 2\cos\Omega + 3\cos^2\Omega$$

4. Consider the following two sequences:

$$\begin{aligned} x_3[0] &= 1 \\ x_3[1] &= 1 \\ x_3[2] &= 2 \\ x_3[3] &= 2 \end{aligned}$$

and $x_3[n] = 0$, otherwise. And

$$\begin{aligned} x_4[0] &= 1 \\ x_4[1] &= 1 \\ x_4[2] &= 2 \\ x_4[3] &= 2 \\ x_4[4] &= 1 \\ x_4[5] &= 1 \\ x_4[6] &= 2 \\ x_4[7] &= 2 \end{aligned}$$

and $x_4[n] = 0$, otherwise. Suppose that we know $X_3(\Omega)$, the DTFT of $x_3[n]$. Find $X_4(\Omega)$, the DTFT of $x_4[n]$, in terms of $X_3(\Omega)$.

5. **Note: This problem is not as hard as you might think at first.**

Let $x[n]$ be a purely real sequence. You work in the laboratory to obtain the following information about $x[n]$:

(i) $x[n]$ is a causal sequence.

(ii) If $v[n] = x[n+2]$, the Fourier transform $V(\Omega)$ of the discrete-time sequence $v[n]$ is purely real.

(iii)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega = 16$$

(iv)

$$x[0] = 2$$

(v)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega}d\Omega = 1.$$

(vi) $x[2] > 0.$

Find the discrete-time sequence $x[n]$.

6. A discrete-time linear and time-invariant (LTI) system is described by:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$$

where $x[n]$ and $y[n]$ are the input and output of the system, respectively.

(a) Determine the frequency response $H(\Omega)$ of the system.

(b) Find the impulse response $h[n]$ of the system.

(c) Find $y[n]$ if $x[n] = 0.5^n u[n]$.