Midterm #2 Solutions

ECE 564/645 — Spring, 2014

1) (a) • ~~$n=7$, $n-k=3 \Rightarrow k=4 \Rightarrow r=4/7$~~

• Look for columns of H that sum to zero (and result in linearly indep rows)

$$G = \begin{bmatrix} 0&0&1&0&0&0&1 \\ 1&1&0&1&0&0&0 \\ 1&1&0&0&1&0&1 \\ 0&1&0&0&0&1&1 \end{bmatrix}$$

• $2^{n-k} = 8$

• d_{min} = # of columns of H that sum to zero $= 2$

$$t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = 0 \;!$$

(b)

$\underline{s}$	coset leader
000	0000000
001	0010000 or 0000001
010	0100000
011	0000010
100	1000000
101	1010000
110	0001000
111	0000100

• $Hy^T = 010$
$\Rightarrow \hat{\underline{e}} = 0100000$
$\Rightarrow \hat{\underline{c}} = 1101000$

• $Hy^T = 001$
$\Rightarrow \hat{\underline{e}} = 0010000$
$\Rightarrow \hat{\underline{c}} = 1101000$

(c) No, the (7,4) Hamming code has $d_{min}=3$

$$H=\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(not unique - any H with distinct columns is fine)

2) (a)

An H with distinct columns

$$H=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(b)

s	leader
0000	000000000
0001	100000000
0010	010000000
0011	001000000
0100	000100000
0101	000010000
0110	000001000
0111	000000100
1000	000000010
1001	000000001
1010	010000010
1011	001000010
1100	000100010
1101	000010010
1110	000001010
1111	000000110

(c)

$$P(E) = 1 - P(c)$$

$$= 1 - \left((1-p)^9 + 9(1-p)^8 p + 6(1-p)^7 p^2\right.$$

(d) No. Would need

$1 + 9 + \binom{9}{2} = 46$ syndromes

(only have 16)

3)

(a) • $k=2,\ n=8 \qquad r = 1/4$

• $G = [I \mid A] \Rightarrow H = [A^T \mid I]$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

• $2^{n-k} = 64$

• $\mathcal{C} = \{00000000, 10111110, 01001111, 11110001\}$

$\Rightarrow d_{min} = 5 \Rightarrow t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = 2$

(b)

$\underline{c}$	$\underline{x}$	$d_E^2(\underline{c}, \underline{d})$
00000000	+1 +1 +1 +1 +1 +1 +1 +1	$4 \cdot 0.9^2 + 2.1^2 + 3 \cdot (0.1)^2$
10111110	−1 +1 −1 −1 −1 −1 −1 +1	big
01001111	+1 −1 +1 +1 −1 −1 −1 −1	big
11110001	−1 −1 −1 −1 +1 +1 +1 −1	$4 \cdot 1.1^2 + 4 \cdot (0.1)^2$ ✓

c) demod: 00100000

$\Rightarrow$ closest to 00000000

d) 11110001

(e)

$$P(E) \le \sum_{i=1}^{3} Q\left(\frac{d_{0,i}}{\sqrt{2N_0}}\right)$$

$$= 2Q\left(\sqrt{\frac{10}{N_0}}\right) + Q\left(\sqrt{\frac{12}{N_0}}\right)$$

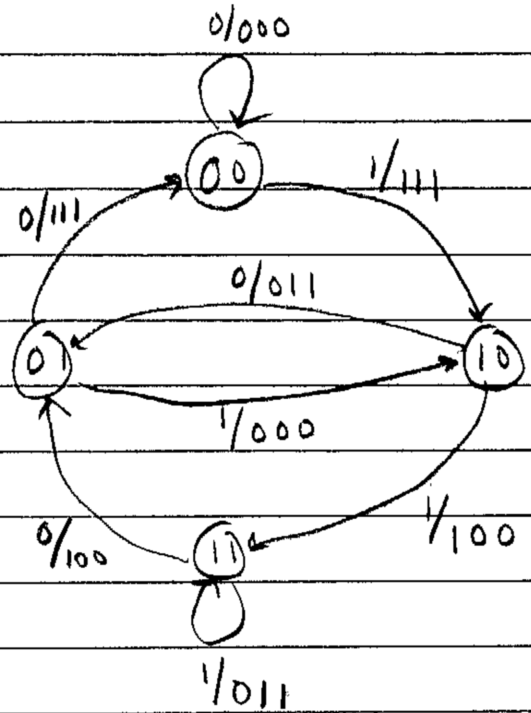
$E_s = 1$

$$= 2Q\left(\sqrt{\frac{10E_s}{N_0}}\right) + Q\left(\sqrt{\frac{12E_s}{N_0}}\right)$$

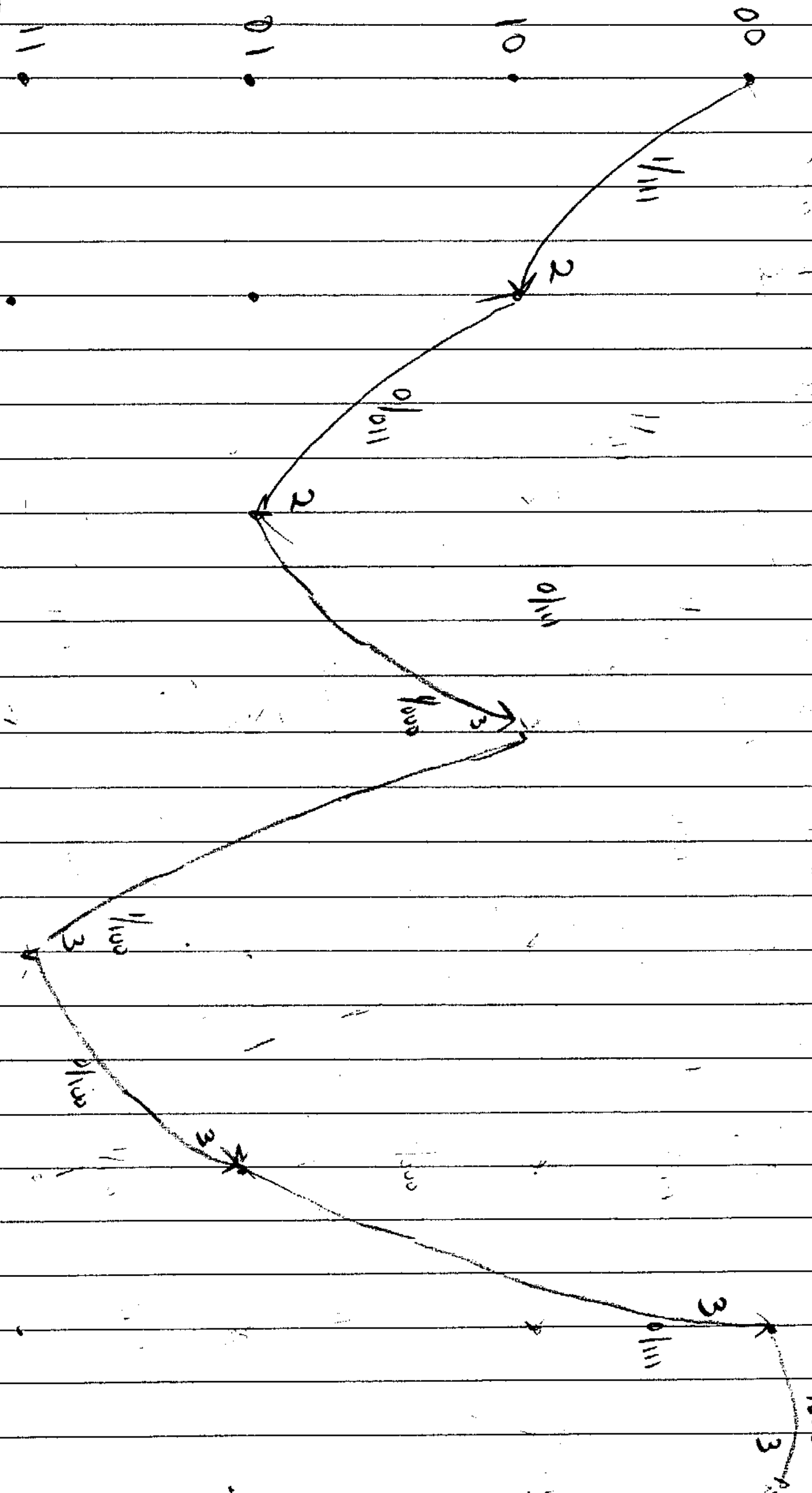
$E_s = \frac{1}{4}E_b$

$$= 2Q\left(\sqrt{\frac{2.5E_b}{N_0}}\right) + Q\left(\sqrt{\frac{3E_b}{N_0}}\right)$$

4) (a)



(b)



OUTPUT: 10110

5) (a)

Would need three codewords of wt 5 or greater. Two tries:

2nd c_i of wt. 6	2nd c_i of wt. 5
000000	000000
111111	111110
X	X

(b)

Need to find an H s. th. each error pattern to be corrected goes into a different syndrome

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(other choices, as well)

s	leader
0000	000000
0001	100000
0010	110000
0011	010000
0100	011000
0101	000010
0110	101000
0111	001000
1000	001100
1001	000001
1010	000110
1011	110001
1100	000011
1101	110100
1110	100100
1111	000100

note: all error patterns I want to correct are in distinct syndromes