Midterm 2 Solutions

ECE 564/645 Spring, 2013

1) (a) This is a (6,2) code:

- $r = k/n = 2/6 = 1/3$
- $\mathcal{C} = \{000000, 101010, 010101, 111111\}$
- $A(x) = 1 + 2x^3 + x^6$, $d_{min} = 3$
- $G = [I_2 \mid A] \Rightarrow H = [A^T \mid I_4]$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- $2^{n-k} = 16$

(b)

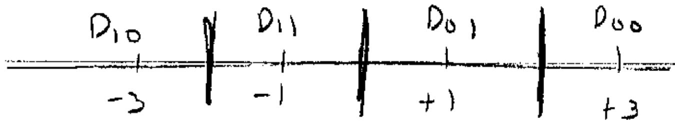
No

$\mathcal{C} = \{000000, 111100, 001111, 111111\}$

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

has $d_{min} = 4$.

2)



(a)

$$\underbrace{\begin{matrix}-1.0, & -2.1, & -2.1, \\ \downarrow & \downarrow & \downarrow \\ 11 & 10 & 10\end{matrix}}\quad \underbrace{\begin{matrix}2.1, & 1.2, & 0.3 \\ \downarrow & \downarrow & \downarrow \\ 00 & 01 & 01\end{matrix}}$$

Hamming distance $\longrightarrow$ closest to 101010   closest to 010101

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$\hat{u} = 10 \qquad\qquad \hat{u} = 01$$

output 1001

(b) <u>think</u>

$$00 \Rightarrow 000000 \Rightarrow (+3, +3, +3)$$
$$10 \Rightarrow 101010 \Rightarrow (-3, -3, -3)$$
$$01 \Rightarrow 010101 \Rightarrow (+1, +1, +1)$$
$$11 \Rightarrow 111111 \Rightarrow (-1, -1, -1)$$

$$\underbrace{-1.0, -2.1, -2.1,}\ \underbrace{1.5, 1.9, 3.5}$$

Euclidean distance $\longrightarrow$ closest to $\underbrace{(-1, -1, -1)}_{11}$ closest to $\underbrace{(+3, +3, +3)}_{00}$

For example:

$$d_E^2(y, c_{00}) = (-1-3)^2 + (-2.1-3)^2 + (-2.1-3)^2$$

c)

$$P(E|\underline{s}_0) \leq \sum_{i=1}^{3} P(\text{choose } \underline{s}_i \mid \underline{s}_0 \text{ sent})$$

$$= \sum_{i=1}^{3} Q\left(\frac{d_E(\underline{s}_i, \underline{s}_0)}{\sqrt{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{108}{2N_0}}\right) + Q\left(\sqrt{\frac{12}{2N_0}}\right) + Q\left(\sqrt{\frac{48}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{6}{N_0}}\right) + Q\left(\sqrt{\frac{24}{N_0}}\right) + Q\left(\sqrt{\frac{54}{N_0}}\right)$$

3)

(a)

Need to find three codewords of wt. ≥ 5 that are still a distance 5 apart.
But any two wt. 5 codewords overlap in at least 3 places:

1111100
0011111

and thus $d_{min} \leq 4$. for a (7,2) code.

(b)

- Start with an (n,k) code: $r = k/n$
  Shortened, it is $(n-1, k-1)$ $r = \frac{k-1}{n-1} = \frac{k}{n} - \frac{1}{n-1}$

  smaller

- Recall

  d_{min} = min # of columns that sum to $\underline{0}$

  removing columns of H can leave this the same or make it larger.

(c)

- All columns are distinct $\Rightarrow d_{min} \geq 2$

but $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$\Rightarrow d_{min} \leq 3$; thus, $d_{min} = 3$

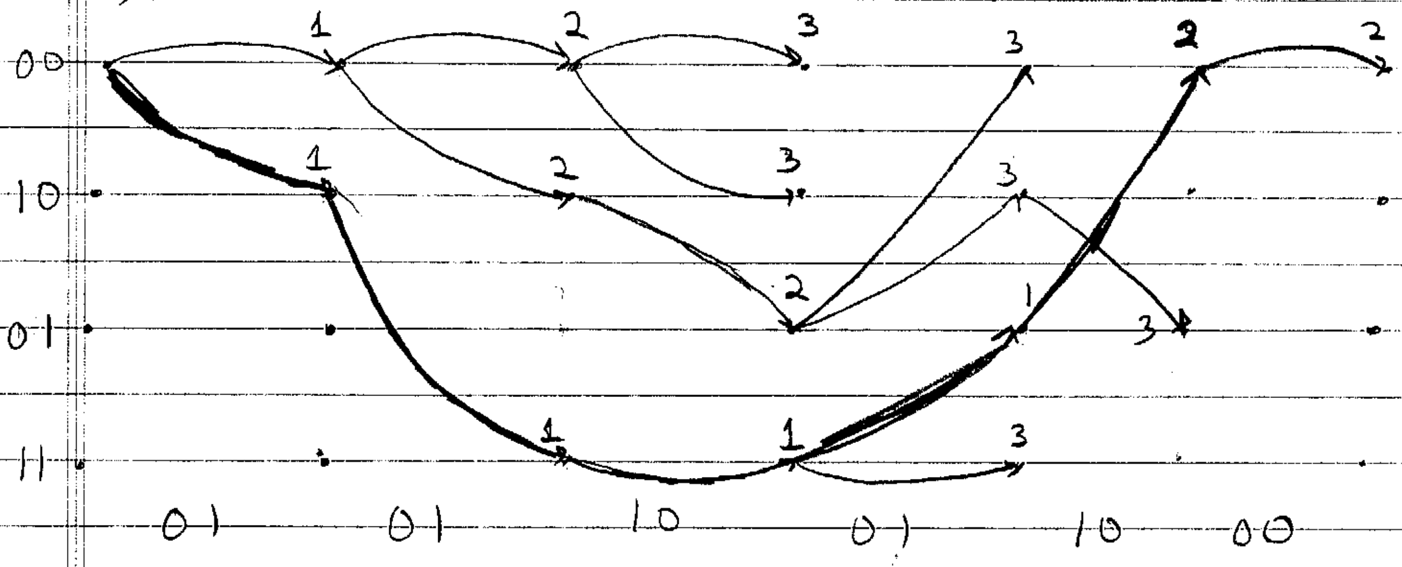
- $2^{n-k} = 2^5 = 32$, which is the right number for the all 0's error pattern and every wt 1 pattern (of which there are 31).

- There is no $(7,2)$, $d_{min} = 5$ code; thus, by part (b), the best I can get is the $(6,1)$ repetition code.

$\mathcal{C} = \{000000, 111111\}$

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

4)



output

1 1 1 0

5) (a)

$$\tilde{G} = H$$

$\Rightarrow$ $\tilde{G}$ is $(n-k) \times n$ $\Rightarrow$ length $= n$

bits codeword $= n-k$

rate $= \frac{n-k}{n} = 1 - k/n = 1 - r$

(b)

$$\underline{x} \oplus \underline{c}_i = \underline{u}^T \tilde{G} \underline{c}_i^T = \underline{u}^T H \underline{c}_i^T = \underline{0}$$

since $H \underline{c}_i^T = 0$

(c) No. $2^k + 2^{n-k} < 2^n$

(d) $\tilde{G} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

$\mathcal{C} = \{0000000, 0001111, 0110011, 1010101,$
$0111100, 1011010, 1100110, 1101001\}$

$$A(x) = 1 + 7x^4$$