ECE 564/645 - Digital Communication Systems (Spring 2014)

Midterm Exam #2

Wednesday, April 16th, 7:00-9:00pm, MARSTON 132

Overview

- The exam consists of four (or five) problems for 100 (or 120) points. The points for each part of each problem are given in brackets - you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. a negative probability), be sure to write "this must be wrong because ..." so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly** - the *minimum penalty* will be an "F" for the course.

1. Consider the following parity check matrix H of a linear block code:

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) Code basics:

- [2] What is the rate r of this code?
- [5] Find a generator matrix G for the code.
- [2] How many syndromes are there?
- [3] How many errors can this code correct? (*Recall that, for a code to be t-error correcting, it must correct* **every** *pattern of t or fewer errors.*)

[8] (b) List the syndromes and their associated coset leaders. **Then,** use them to decode the following received vectors (when I say "decode" here, I mean to give the length-7 codeword that is closest to the received vector):

- 1001000
- 1111000

[5] (c) Does this code have the best minimum distance of any linear block code with this k and n? If so, prove it. If not, find a better code (give G, H, **or** the codewords).

2. Consider a single error correcting (SEC) $(9, 5)$ linear block code:

[8] (a) Design such a code. Give G, H, **or** the codewords (your choice!).

[7] (b) Write down the syndromes and their associated coset leaders.

[5] (c) Suppose this code is used across a binary symmetric channel with crossover probability p. Find the probability of error $P(E)$ of the code (i.e. the probability that the wrong codeword is chosen at the receiver).

[5] (d) Does there exist a double error correcting (DEC) $(9, 5)$ linear block code?

3. Consider the following generator matrix G of a linear block code:

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Code basics:

- [2] What is the rate r of this code?
- [3] Find the parity check matrix H for the code.
- [5] How many errors can this code correct? (*Recall that, for a code to be t-error correcting, it must correct* **every** *error pattern of t or fewer errors.*)

[7] (b) Supppose the output of the coder is mapped to binary phase-shift keying (BPSK) with the mapping $0 \to +1$, $1 \to -1$. The resulting bits are transmitted across an additive white Gaussian noise channel where the noise has power spectral density $S_n(f) = \frac{N_0}{2}$. Hard-decision decode the

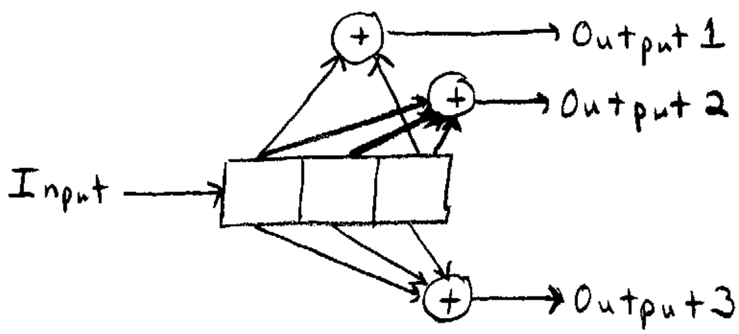
following vector; that is, the demodulator converts each element of the vector to a 0 or 1 using an optimum BPSK demodulator, and then the decoder takes the resulting binary string and finds the closest codeword (when I say "decode" here, I mean to give the length-7 codeword that is closest to the received vector):

0.1, 0.1, -1.1, 0.1, 0.9, 0.9, 0.9, 0.1

[8] (c) Supppose the output of the coder is mapped to binary phase-shift keying (BPSK) with the mapping $0 \to +1$, $1 \to -1$. The resulting bits are transmitted across an additive white Gaussian noise channel where the noise has power spectral density $S_n(f) = \frac{N_0}{2}$. Soft-decision decode the following vector; that is, find the codeword that the optimum demodulator/decoder would produce (when I say "decode" here, I mean to give the length-7 codeword that is closest to the received vector):

0.1, 0.1, -1.1, 0.1, 0.9, 0.9, 0.9, 0.1

4. Consider the following rate-1/3, memory 2 shift register circuit for generating a convolutional code:



[10] (a) Find the state diagram for this convolutional code.

[10] (b) Use the Viterbi algorithm to decode the following received bit sequence (assume we start in the "00" state, and that the last two input bits are forced to "0" to drive it back to the "00" state; thus, the input is five information bits **which is what you are trying to figure out** - and then "0", "0"):

001011001100100111000

5. Design of an interesting $(6, 2)$ code:

[5] (a) Show that there is no $(6, 2)$ code that is double-error correcting. (*Recall that, for a code to be double-error correcting, it must correct* **every** *error pattern of* 2 *or fewer errors.*)

[ECE 645 - 15 points, ECE 564 - 5 BONUS] (b) Suppose the only error patterns that ever occur on your channel have either: (a) no errors, (b) a single error, or (c) two *adjacent* errors. Stated differently, the possible error patterns are 000000, 100000, 010000, 001000, 000100, 000010, 000001, 110000, 011000, 001100, 000110, 000011.

- Find a (6,2) linear block code that corrects these error patterns (Give H, G, or the codewords).
- List the syndromes and their associated coset leader (i.e. the error pattern you will correct when you get that syndrome.)