ECE 313 - Signals and Systems, Fall 2012

Midterm Exam #2

Wednesday, November 14th, 7:00-9:00pm, ELABII 119

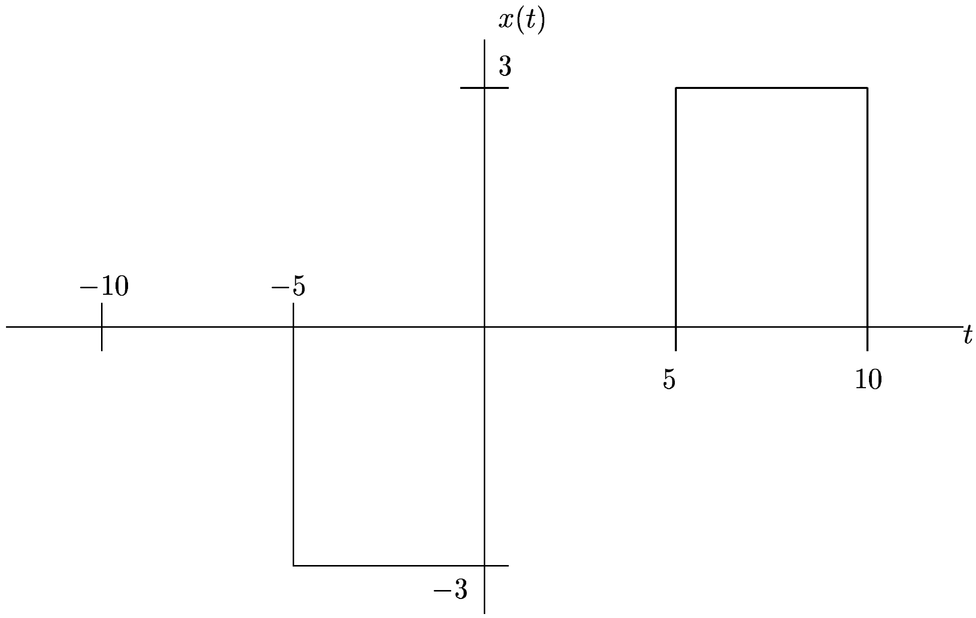
Overview

- The exam consists of four problems for 100 points. The points for each part of each problem are given in brackets **-** you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **two page-sides** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write “this must be wrong because . . . ” so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an “F” for the course.

1. The signal $x(t)$ is shown below:



[5] (a) Write $x(t)$ in terms of the "rectangle function" rect(t).

[5] (b) Find the Fourier transform $X(f)$ of $x(t)$.

[8] (c) Find the magnitude squared $|X(f)|^2$ of $X(f)$. Since we know $|X(f)|^2$ is real, simplify your answer at least until all imaginary components (i.e. $j = \sqrt{-1}$) are gone.

[7] (d) Suppose $x(t)$ is input to a linear time-invariant (LTI) system with impulse response $h(t) =$ rect$(t/5)$ to arrive at the output $y(t)$. Find $H(f), Y(f)$ and the output $y(t)$. (*Hint: Many of you will be able to check your answer for $y(t)$ by quickly calculating $y(t) = h(t) * x(t)$ graphically.*)

2. The input $x(t) = 5\text{sinc}^2(20(t-5))$ is input to a linear time-invariant (LTI) filter with impulse response $h(t) = 5\text{sinc}(10t)$ to produce the output $y(t)$.

   [10] (a) Find $X(f)$. **Plot its magnitude and phase.**

   [5] (b) Find $Y(f)$ and **plot its magnitude**.

   [5] (c) The "energy gain" of a filter is defined as:

$$E_h = \int_{-\infty}^{\infty} |h(t)|^2 dt$$

   Find the energy gain of the filter $h(t) = 5\text{sinc}(10t)$.

3. *DT Systems questions:*

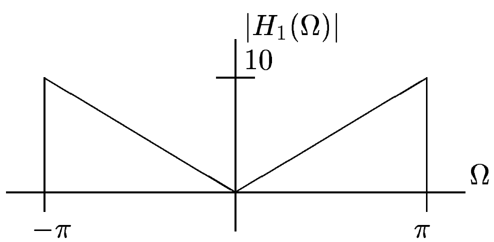
   [10] (a) Suppose that the input $x_a[n] = \text{rect}((n-1)/3)$ is input to a linear and time-invariant (LTI) discrete-time system with impulse response $h_a[n] = \Lambda(n/3)$. Find the output $y_a[n]$. (*Hint: Do this in the time domain.*)

   [5] (b) Suppose that the input $x_b[n] = 3\text{rect}((n-1)/3)$ is input to a linear and time-invariant (LTI) discrete-time system with impulse response $h_b[n] = 2\Lambda(n/3) + 3\Lambda(n/3 - 1/3)$. Find the output $y_b[n]$ in terms of $y_a[n]$ from part (a).

   [15] (c) Consider the linear time-invariant (LTI) discrete-time system defined by the equation $y_c[n] = -y_c[n-1] + x_c[n]$, where $x_c[n]$ is the system input and $y_c[n]$ is the system output.

   - Find the frequency response $H(\Omega)$ of this system.
   - *Roughly* plot $|H(\Omega)|^2$.
   - Using $|H(\Omega)|^2$ as a guide, find a bounded input $x_c[n]$ that leads to an unbounded output $y_c[n]$, thus showing that the system is not stable.

4. Consider the magnitude and phase of the frequency response $H_1(\Omega)$ of a linear and time-invariant (LTI) discrete-time System 1, given for $-\pi \leq \Omega \leq \pi$, as:



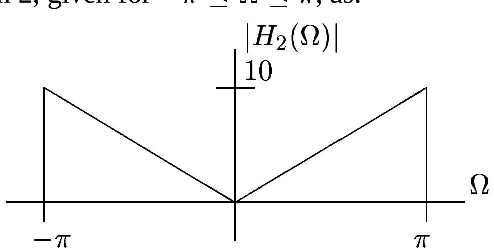
phase $H_1(\Omega) = 0$ for all Ω

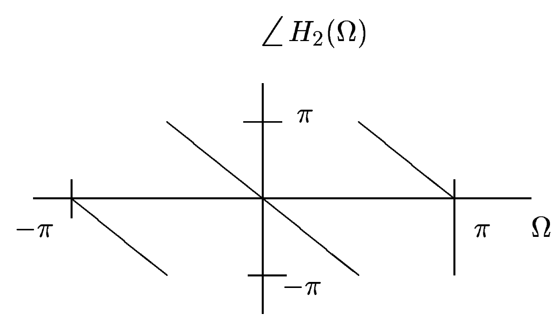
   [5] (a) Suppose $x_a[n] = 5\cos(\frac{\pi}{3}n)$ is input to System 1. Find the output $y_a[n]$.

   [5] (b) Suppose $x_b[n] = 5\cos(\frac{4\pi}{3}n)$ is input to System 1. Find the output $y_b[n]$.

   [5] (c) Suppose I take the discrete-time signal from part (a): $x_a[n] = 5\cos(\frac{\pi}{3}n)$, but I remove half of the values: to arrive at a new signal $x_c[n] = x_a[2n]$. I now input $x_c[n]$ into System 1 and get the output $y_c[n]$. Find $y_c[n]$.

   Next, consider the frequency response $H_2(\Omega)$ of a linear and time-invariant (LTI) discrete-time System 2, given for $-\pi \leq \Omega \leq \pi$, as:





   [5] (d) Suppose $x_d[n] = 5\cos(\frac{\pi}{3}n)$ is input to System 2. Find the output $y_d[n]$. (*Hint: If you can get part (e), it might give you a different way to solve this part.*).

   [5] (e) Suppose the impulse response of System 1 is given by $h_1[n]$. Write $h_2[n]$, the impulse response of System 2, in terms of $h_1[n]$.