Overview

- The exam consists of five problems for 115 points. The points for each part of each problem are given in brackets - you should spend your two hours accordingly.

- The exam is closed book, but you are allowed one page-side of notes. Calculators are not allowed. I will provide all necessary blank paper.

Testmanship

- Full credit will be given only to fully justified answers.

- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.

- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.

- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.

- If you get to the end of the problem and realize that your answer must be wrong, be sure to write “this must be wrong because . . .” so that I will know you recognized such a fact.

- Academic dishonesty will be dealt with harshly - the minimum penalty will be an “F” for the course.
1. Filters play a critical role in transmitters and receivers. For example, a simple linear time-invariant (LTI) $RC$-filter is part of the conventional AM receiver. Consider such a filter with impulse response:

$$h(t) = \frac{1}{RC}e^{-t/RC}u(t)$$

where $R$ is the value of the resistor (in ohms), $C$ is the value of the capacitor (in farads), and $u(t)$ is defined in the standard way:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{else} \end{cases}$$

The frequency response of the filter is found by taking its Fourier transform of $h(t)$ to yield:

$$H(f) = \frac{1}{1 + j2\pi fRC}$$

[8] (a) Suppose $R = 20 \, k\Omega$ and $C = 5 \, \mu F$. Find and roughly sketch $|H(f)|^2$. For your sketch, draw the rough shape and indicate the frequency values for which $|H(f)|^2$ goes through a couple of levels (for example: you could indicate the $f$ on the x-axis for which $|H(f)|^2 = \frac{1}{4}$ and the $f$ on the x-axis for which $|H(f)|^2 = \frac{1}{2}$).

[7] (b) The signal $x(t) = 5\cos(2\pi 2t)$ is input to your $RC$-filter with $R = 20 \, k\Omega$ and $C = 5 \, \mu F$, as in part (a). Find the power at the output of the filter.

[5] (c) Your boss is unhappy about the power coming out of the filter when the sinusoid is input in part (b) and wants you to reduce that power (i.e. the power at the output when $x(t) = 5\cos(2\pi 2t)$ is input) by roughly 50% (i.e. cut it in half) without reducing $|H(0)|$. Find new values of $R$ and $C$ that accomplish such a reduction. (Hint: Since you do not have a calculator, you are going to have to approximate some values here. That is fine.)
2. Suppose that we have a bandpass signal $x(t)$ of bandwidth $2W$ around carrier frequency $f_c$, which we desire to filter with the following bandpass filter:

$$h(t) = W \text{sinc}(Wt) \cos \left(2\pi f_c t - \frac{\pi}{4}\right) + 2W \text{sinc}(2Wt) \cos \left(2\pi f_c t + \frac{\pi}{4}\right)$$

As we know from class, building *bandpass* filters is hard so we desire instead to do our filtering with *lowpass* filters by completing the following steps.

8. (a) Find the in-phase ($h_I(t)$) and quadrature ($h_Q(t)$) components of $h(t)$; that is, find real, lowpass signals $h_I(t)$ and $h_Q(t)$ such that $h(t) = h_I(t) \cos(2\pi f_c t) - h_Q(t) \sin(2\pi f_c t)$.

7. (b) Find and *roughly* sketch the frequency responses $H_I(f)$ and $H_Q(f)$.

7. (c) Draw a block diagram of a system with input $x(t)$ and output $h(t) \ast x(t)$ that uses only summers, multipliers, oscillators, and *lowpass* filters (i.e. filters that are lowpass with bandwidth $W$). Be sure to give the frequency response of all filters. Remember that all signals and filter impulse responses must be real, of course.

8. (d) Find $y(t) = h(t) \ast x(t)$ if

$$x(t) = \cos \left(\frac{3W}{4}t\right) \cos(2\pi f_c t)$$

3. [10] (a) I have a square-law device with $y(t) = a_0 + a_1 x(t) + a_2 x^2(t)$ output when $x(t)$ is input, where $a_0$, $a_1$, and $a_2$ are positive constants. Suppose I also have the capability to design an ideal filter of any sort. Show how I can use the square-law device and linear time-invariant (LTI) filtering to build a circuit that doubles the frequency of an input sinusoid of known frequency $f_1$.

10. (b) Design a circuit that accepts two inputs:

$$\cos(2\pi f_0 t) \quad \text{and} \quad \cos(2\pi(f_0 + \Delta f) t + \theta)$$

and outputs a DC voltage that is proportional to the frequency difference $\delta f$. Note that $f_0$ and $\theta$ are unknown (as, of course, is $\Delta f$), but you can assume that $\Delta f \ll f_0$. You can use any of the standard components from class: multipliers, summers, envelope detectors, filters of any type, etc.
Problems 4 and 5

On the next page are plots of two signals.

For each of the signals:

If the signal is an AM signal:

- [5] Assuming the message \( m(t) \) has no DC component, give a possible message signal \( m(t) \).
- [5] Assuming the message \( m(t) \) has no DC component, give the modulation index of the system.
- [10] Assuming that \( x(t) \) continues infinitely in time in both directions with the same pattern, sketch \( |X(f)| \), the magnitude of the Fourier transform of the signal \( x(t) \). **What is the bandwidth of the signal \( x(t) \)?**
- [5] Give a detailed circuit using only basic components (resistors, capacitors, inductors, diodes, etc.) with input \( x(t) \) and output \( m(t) \). Be sure to specify important component values.

If the signal is an FM signal:

- [7] Give the message signal \( m(t) \) if the carrier frequency is 3.5 Hz and the frequency deviation constant is \( k_f = 0.5 \) Hz/volt. (Hint: The message changes at most once per second.)
- [8] Assuming that the bandwidth of the rectangular pulse \( p(t) \) can be approximated as 4 Hz, find the modulation index \( \beta \).
- [5] Use Carson’s Rule to estimate the bandwidth of the signal.
Figure 1: Signal $x(t)$ for Problem 4.

Figure 2: Signal $x(t)$ for Problem 5.