Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets - you should spend your two hours accordingly.

- The exam is closed book, but you are allowed one page-side of notes. Calculators are not allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**

- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.

- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.

- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.

- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write “this must be wrong because . . . ” so that I will know you recognized such a fact.

- Academic dishonesty will be dealt with harshly - the minimum penalty will be an “F” for the course.
Some potentially useful information

\[
\begin{align*}
\cos(\theta) &= \frac{1}{2} \left( e^{j\theta} + e^{-j\theta} \right) \\
\sin(\theta) &= \frac{1}{2j} \left( e^{j\theta} - e^{-j\theta} \right)
\end{align*}
\]

\[
\begin{align*}
\sin(a \pm b) &= \sin(a) \cos(b) \pm \cos(a) \sin(b) \\
\cos(a \pm b) &= \cos(a) \cos(b) \mp \sin(a) \sin(b)
\end{align*}
\]

\[
\begin{align*}
\cos(a) \cos(b) &= \frac{1}{2} [\cos(a - b) + \cos(a + b)] \\
\sin(a) \sin(b) &= \frac{1}{2} [\cos(a - b) - \cos(a + b)] \\
\sin(a) \cos(b) &= \frac{1}{2} [\sin(a - b) + \sin(a + b)]
\end{align*}
\]

<table>
<thead>
<tr>
<th>Time Function</th>
<th>Fourier Transform</th>
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<td>( x(at + b) )</td>
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| \( p(t) = \begin{cases} 
1 & |t| \leq 1/2 \\
0 & |t| > 1/2 \end{cases} \) | \( P(f) = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f) \) |
| \( \cos(2\pi f_0 t) \) | \( \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)] \) |
| \( \sin(2\pi f_0 t) \) | \( \frac{1}{2j} \delta(f - f_0) - \frac{1}{2j} \delta(f + f_0) \) |
| \( x(t) = \begin{cases} 
1 - |t| & |t| \leq 1 \\
0 & |t| > 1 \end{cases} \) | \( X(f) = \text{sinc}^2(f) \) |

Parseval’s Relation: If \( X(f) \) is the Fourier Transform of \( x(t) \),

\[
\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df
\]
1. We know from class that a bandpass communication signal $x(t)$, which has Fourier transform $X(f)$ that is non-zero only for $f_c - W \leq |f| \leq f_c + W$, where $W$ is a positive constant, can be written as

$$x(t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t)$$

where $x_I(t)$ and $x_Q(t)$ are real, lowpass signals of bandwidth $W$.

[15] (a) Show that the system shown below produces $x_I(t)$, as indicated (no need to show $x_Q(t)$). Be sure to precisely give all of the steps (no short cuts or hand-waving), and note which properties of the Fourier transform you employ, if necessary.

![Diagram of a bandpass system with a lowpass filter (LPF) and 2cos(2πfct) and 2sin(2πfct) inputs.]

[5] (b) Now, suppose that:

$$x(t) = 500 \text{sinc}^2(200t) \cos(2\pi f_c t + \frac{\pi}{4})$$

Find $x_I(t)$ and $x_Q(t)$, the outputs of the circuit from (a).

2. A sinusoidal message signal of the form $m(t) = A_m \cos(2\pi f_m t + \theta)$ is amplitude modulated on a carrier of frequency $f_c = 1$ MHz to form the DSB-SC signal $x(t) = m(t) \cos(2\pi f_c t)$, whose Fourier transform $X(f)$ is shown below.

[8] (a) Find $A_m$, $f_m$, and $\theta$.

[7] (b) Find the power in the signal $x(t)$.

[5] (c) Because we desire the simple conventional AM receiver, we add a carrier component $A_c \cos(2\pi f_c t)$ to $x(t)$. Find the smallest value of $A_c$ such that a conventional AM receiver will work.

[5] (d) Recall that the power efficiency of an AM system is the ratio of the power invested in the part of the signal carrying the message and the total signal power. Find the power efficiency of your system from (c).
3. **Power and Energy:**

[8] (a) Find the energy in the signal $x(t) = 50 \text{sinc}(t/50)$.

[10] (b) Find the power in the signal $x(t) = 4 \cos^2(2\pi \ell)$.

[7] (c) Recall that we find the Hilbert transform $\hat{x}(t)$ of a signal $x(t)$ by running $x(t)$ through a linear time-invariant (LTI) filter $h(t)$ with frequency response:

$$H(f) = \begin{cases} \frac{-j}{f}, & f \geq 0 \\ \frac{j}{f}, & f < 0 \end{cases}$$

How does the energy of $\hat{x}(t)$ compare to that of $x(t)$? (*Be sure to justify your answer, of course.*)

4. [10] The diode in the conventional AM receiver gives us a pretty clear indication that this receiver is not a linear time-invariant (LTI) system. If we ignore receiver complexity constraints, does there exist a linear time-invariant (LTI) system that can be used as a receiver for conventional AM radio?

5. Your new boss asks you to design a bandpass filter to be applied to inputs $y(t)$ of bandwidth 10 kHz around a carrier $f_c$. The filter response that she wants is given by:

![Filter Response Diagram]

However, your group is not good at designing bandpass filters, so your boss tells you “Convert $y(t)$ to baseband, apply I/Q filtering, and then convert it back up to bandpass.” Design such a system in two steps:

[10] (a) Finding $H_I(f)$ and $H_Q(f)$. *Hint:* Recall from class that

$$H_I(f) = \frac{H_Z(f) + H_Z^*(-f)}{2}$$

$$H_Q(f) = \frac{H_Z(f) - H_Z^*(-f)}{2j},$$

where $H_Z(f)$ is the Fourier transform of the complex envelope of $h(t)$.

[10] (b) Draw the block diagram of your system that takes as input $y(t)$ and produces output $h(t) * y(t)$, per the instructions of your boss above. Be sure to draw the simplest block diagram (i.e. do not leave a block diagram that has extraneous blocks in it) and provide all filter responses (impulse response or frequency response is fine), etc.